TÉCNICO LISBOA

Computational Experiments on Maintenance Scheduling in Airline Companies

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Abstract

Essential for complying with the strict safety regulations in the aviation industry and with the increasing passengers demand, aircraft maintenance planning has become a factor of the upmost importance for operational efficiency and cost optimization, which is crucial for the airlines to face fierce global competition. An effective method to optimize aircraft maintenance operations is to minimize the associated costs, by reducing the amount of maintenance activities, and consequently, increasing aircraft availability. Thus, a mixed-integer linear programming model and a heuristic approach are presented, which minimizes aircraft maintenance costs. This mathematical optimization model creates a maintenance schedule, including light maintenance checks (A-type) and heavy maintenance checks (C-type), during a specified planning horizon. Firstly, the model is verified by applying it to an illustrative example, showing the applicability of both the branch-and-bound and the heuristic approaches. Then, both approaches are applied to a case study of the “narrow-body” aircraft fleet of the Portuguese airline, TAP Air Portugal, for a two-year planning horizon. The results show that with the heuristic approach the computational time can be reduced to 48 minutes, while providing equal or lower maintenance costs than the branch-and-bound approach that showed non-zero optimality gaps. Finally, some sensitivity analysis associated with threshold values, the COVID-19 pandemic situation and the hangar capacity availability, are studied. Overall, this work provides a decision framework that can support aircraft maintenance planning, while reducing the planning time and providing near-optimal feasible solutions.

Key Words: Maintenance, Maintenance Scheduling, Aircraft Maintenance, Mixed-Integer Linear Programming, Heuristic

Resumo

Essencial para o cumprimento das rígidas normas de segurança do setor aeronáutico e com a crescente demanda de passageiros, o planeamento da manutenção das aeronaves tornou-se um fator de extrema importância para a eficiência operacional e otimização de custos, fundamental para as companhias aéreas enfrentarem a aguerrida concorrência global. Um método eficaz para aumentar os lucros e otimizar as operações de manutenção de aeronaves é minimizar os custos associados, reduzindo a quantidade de atividades de manutenção e, consequentemente, aumentando a disponibilidade das aeronaves. Nesta dissertação, um modelo de programação linear inteira mista e uma abordagem heurística são apresentados, que minimizam os custos de manutenção de aeronaves. Este modelo de otimização matemática cria um calendário de manutenção, incluindo verificações de manutenção leve (tipo A) e verificações de manutenção pesada (tipo C), durante um horizonte de planeamento especificado. Ambas as abordagens são aplicadas a um caso de estudo da frota de aeronaves “narrow-body” da companhia aérea portuguesa, TAP Air Portugal, para um horizonte de planeamento de dois anos. A análise dos resultados demonstrou que a abordagem heurística consegue reduzir o tempo computacional do problema para 48 minutos, ao mesmo tempo que apresenta custos de manutenção iguais ou menores do que a abordagem ramificar e limitar que apresenta um gap de otimalidade não nulo. Finalmente, algumas análises de sensibilidade são estudadas. Em conclusão, esta dissertação fornece uma estrutura de decisão que pode apoiar o planeamento de manutenção de aeronaves, enquanto reduz o tempo de planeamento e fornece soluções otimizadas viáveis.

Palavras Chave: Manutenção, Calendarização de manutenção, Manutenção de aeronaves, Programação linear inteira mista, Heurística

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List of Acronyms

EC	European Commission
EU	European Union
EASA	European Aviation Safety Agency
GDP	Gross Domestic Product
SES	Single European Sky
ATM	Air Traffic Management
SESAR	Single European Sky ATM Research
MPD	Maintenance Planning Document
MRO	Maintenance, Repair and Overhaul
MILP	Mixed-Integer Linear Programming
MIP	Mixed-Integer Programming
ILP	Integer Linear Programming

1 Introduction

In this first section, a brief contextualization, and an introduction to the research topic of the study are presented. Firstly, it is approached the topic of air mobility both in Europe and Portugal, and then the maintenance checks in aircrafts are presented. Secondly, the research problem, the objective of this dissertation and the methodology are described. Finally, the document's structure is provided.

1.1 Context

1.1.1 Air mobility in Europe

Since 1992, air travel in Europe has been revolutionized with the creation of the EU Internal Market for Aviation, which replaced some national rules by a single set of EU rules. Before the creation of the EU Internal Market, prices were high, mainly because competition was low, due to national restrictions, but when those changes took place, barriers have been removed and consequently competition has increased. This meant more airlines stepping forward, more routes and more airports, making air travelling accessible to a larger number of people, which directly impacts the increase in the number of passengers, as seen in Figure 1.1.

Figure 1.1 - Increase of the number of passengers in Europe (Source: EC)

Since then, air travel has not only become cheaper, but also safer. In 2002, the EASA (European Aviation Safety Agency) was founded to become the cornerstone of the European aviation safety system, being responsible for the certification and regulation of all high safety technical parameters in the EU.

All airlines are obligated to follow the strict safety measures to be able to fly in EU airspace, since the EC with the assistance of EASA carries out regular inspections to ensure proper implementation of the rules. Not complying with the safety regulations can bring penalties or suspension of certificates on certificate holders throughout all the Member States of the EU (EC, 2021), which means that being efficient on safety procedures and measures is crucial for the airlines to remain competitive and to reach higher profits.

The increase of safety in air travelling and the decrease of ticket prices resulted in economic growth and in the creation of more jobs, for example, in 2004, aviation contributed over 621 billion euros to EU GDP, while supporting 8.8 million jobs in the EU (EC, 2017). The aviation sector is of the upmost importance since it helps boost Europe global presence by driving trade and tourism, as shown in Figure 1.2.

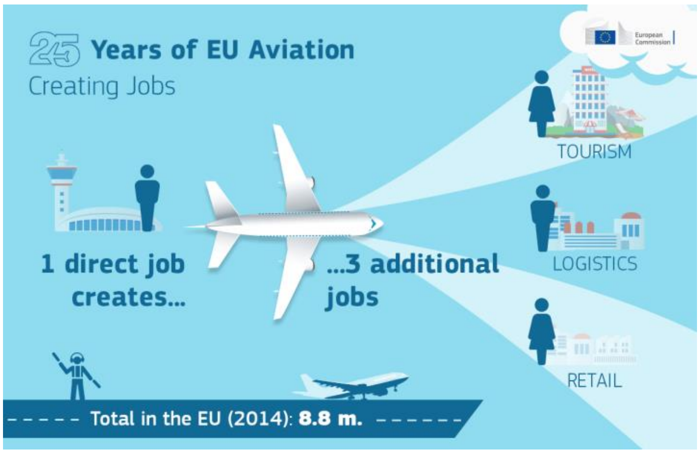


Figure 1.2 - Job Creation from the aviation sector (Source: EC)

In 2004, the initiative for a Single European Sky (SES) was launched, mainly to regulate air traffic management (ATM), because of air traffic growth, but also to regulate cost efficiency and environmental issues. “The EU’s main objective is to reform ATM in Europe in order to cope with sustained air traffic growth and operations under the safest, most cost and flight-efficient and environmentally friendly conditions.” (EC, 2021). To achieve these objectives, besides having a programme called SESAR (Single European Sky ATM Research), new aircraft technologies, cleaner fuels and renewal of the fleet over time were also implemented to achieve a more efficient ATM, which resulted into lower emissions per km flown compared to 1992 (EC, 2021), as Figure 1.3 shows.

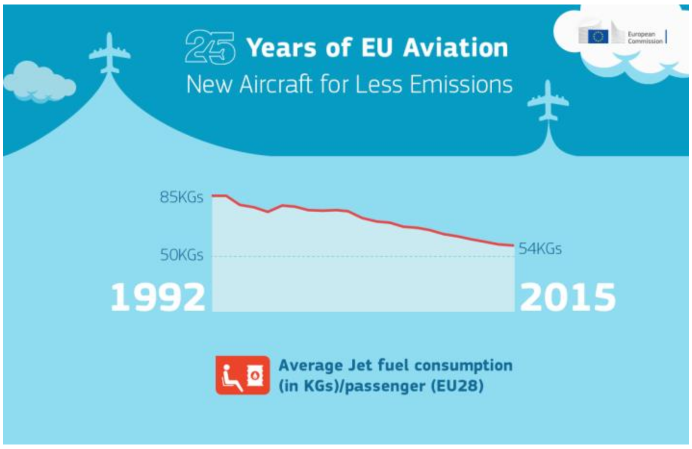


Figure 1.3 - CO2 emissions decrease between 1992 and 2015 (Source: EC)

It is projected that European air traffic will increase 50% by 2035 (EC 2021). This increase in air traffic will make operational efficiency one of the most important aspects that European airline companies need to worry about, making fleet availability and maintenance operations key factors for these companies to succeed, in the face of the growing number of challenges and fierce global competition.

1.1.2 Air mobility in Portugal

Just like any other country in Europe, airline companies are particularly important for tourism and Portugal is no exception, with its capital, Lisbon, as an important hub that serves as a European Gateway.

The major airline company in Portugal is TAP Air Portugal (Transportadora Aérea Portuguesa) and it was founded by Humberto Delgado on 14th of March 1945, at the time with the name of “Secção de Transportes Aéreos”. On the 19th of September 1946, the inaugural flight was made from Lisbon to Madrid with a Dakota DC-3, which was a military plane adapted to civil aviation, becoming this connection the first commercial route for the airline company (Source: TAP company). Since then, TAP Air Portugal never stopped growing, adding more European, intercontinental, and domestic routes, and increasing the fleet numbers.

TAP Air Portugal is part of a major group called TAP Group (TAP, SGPS, S.A.), which also has participation in other companies that offer a wide range of services, such as catering for flights, airport logistics or aircraft maintenance, to TAP Air Portugal mainly, but also to other airline companies. Most recently, on the 16th of July 2020, due to the impacts of the COVID-19 pandemic, a new agreement was announced for a restructuring of shareholders, which involved a Portuguese government financial aid approved by the EC. The present structure of TAP Group is represented on a chart, shown in Figure 1.4.

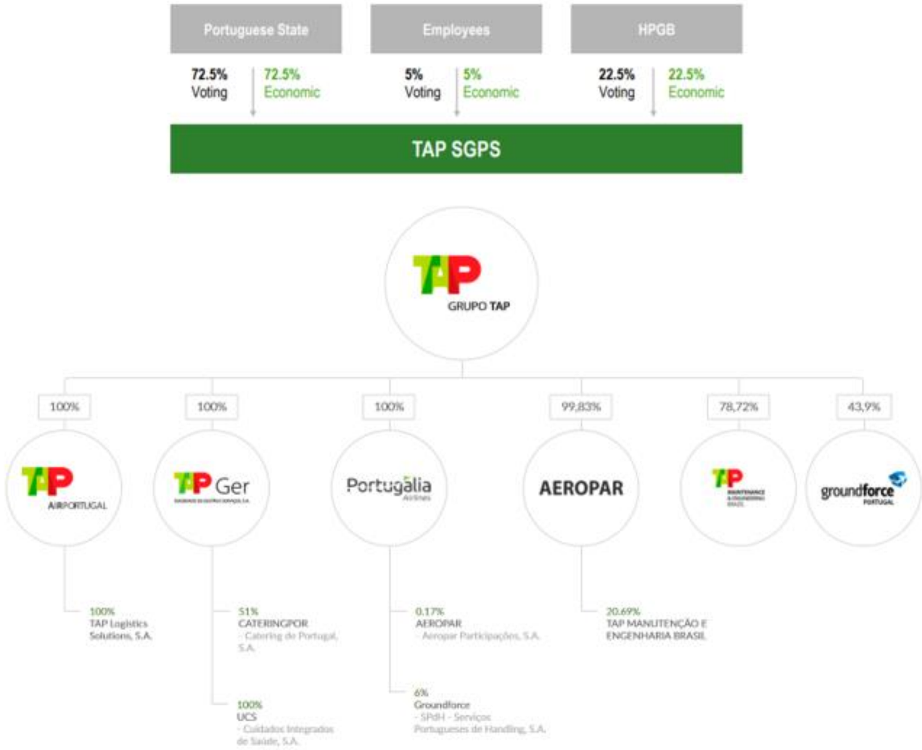


Figure 1.4 - TAP Group (TAP, SGPS, S.S.) present structure [adapted from: TAP company]

To reach energetic, environment and operational efficiency, TAP Air Portugal has been investing in the Airbus neo family aircrafts to integrate their fleet. TAP Group (TAP, SGPS, S.A.) fleet has 86 operating aircrafts that are operated by TAP Air Portugal, shown in Table 1.1, plus 9 Embraer 190 aircrafts that are operated by Portugália Airlines and 8 ATR 72-600 aircrafts that are operated by White Airways. The Airbus aircrafts are divided into two types: the "narrow-body" type (A321, A320 and A319) and the "wide-body" type (A340 and A330) (Source: TAP company).

These aircrafts are responsible for operating flights to 87 different destinations, across Europe, Africa, South America, and North America.

Table 1.1 – Current TAP Air Portugal fleet technical characteristics (Source: TAP company)

Aircraft Type	A330-900 neo	A321-200 LR	A321-200 neo	A320-200 neo	A330-200	A321-200	A320-200	A319-100	Embraer 195
Number of aircrafts	19	10	10	11	5	3	16	8	4
Passengers' capacity [people]	298	168	216	174	263 / 269	216	174	144	118
Maximum range [km]	12,000	7,400	6,000	6,500	12,000	4,600	5,500	5,700	4,260
Fuel Capacity [l]	139,090	32,940	23,580	23,724	139,090	23,700	23,859	23,859	12,971
Cruising speed [km/h]	930	900	900	900	930	900	900	900	870
Cruising altitude [m]	12,500	11,900	11,900	11,900	12,500	11,900	11,900	11,900	12,500
Length [m]	63.69	44.51	44.51	35.57	58.82	44.51	37.57	33.84	38.65
Wingspan [m]	64.00	35.80	35.80	35.80	60.30	34.10	34.10	34.10	28.72
Height [m]	16.79	11.76	11.76	12.08	17.39	12.09	12.14	12.17	10.55
Wing area [m^2]	361.63	122.40	122.40	122.40	361.60	122.40	122.40	122.40	92.50

1.1.3 Aircraft maintenance

The maintenance of aircrafts is one of the most important factors for air safety, mainly because it ensures the airworthiness of the aircrafts, but also because it has a direct impact on air traffic management, since it affects aircraft's availability, as the maintenance activities require to remove the aircrafts out of service. This unavailability has an indirect impact on the profits of the airline company, emphasizing the importance of properly optimizing the scheduling of the maintenance activities in order to reduce costs and minimize unavailability of the airline fleet.

Aircraft maintenance consists mostly of preventive maintenance, since almost all the maintenance activities are done before they reach a certain threshold value of flight hours, flight cycles or time since their last type of maintenance. As preventive maintenance tasks consist of inspecting certain subsystems, these maintenance tasks are usually called checks or inspections.

The Maintenance Planning Document (MPD) is the document that regulates the threshold values to trigger a maintenance check and describes all the maintenance activities that must be done in each check according to the manufacturer (e.g., Airbus). The different types of aircraft maintenance checks are divided into 4 categories and each one is identified by a letter: the A-check, the B-check, the C-check and the D-check. Moreover, each type of check is usually divided into groups of tasks that are numbered, for example the A-check has 4 groups of tasks (A1, A2, A3 and A4) and the C-check has 12 groups of tasks (C1, …, C12).

The A-check is the most basic check and is responsible for tasks like general (visual) inspections for evidence of damage or missing parts, on the interior and the aircraft hull. It can also involve inspections to the engine. This check type usually requires 1 to 2 days to be completed.

The B-check involves tasks such as inspecting the wheel well hydraulic tubing or checking the alignment and torquing of the nose landing gear spotlight. Some tasks of the B-check have been merged into A-check tasks. This check type takes up to 3 days of maintenance time.

The C-check is classified as a heavy-maintenance check. This check requires an aviation maintenance technician, who is responsible to inspect most of the aircraft´s parts, such as the examination of structures for damage or an in-depth lubrification of all fittings and cables. This check type usually takes 1 to 2 weeks to be completed.

The D-check is the heaviest maintenance check since it consists in stripping down the entire aircraft and remove equipment to inspect for corrosion or damage. This check, due to the amount of work it takes to perform the tasks, requires 4 to 6 weeks to be completed, and occurs every 6 to 10 years depending on the aircraft (NAA, 2020).

The aircraft maintenance and engineering unit of TAP Air Portugal is TAP Maintenance & Engineering (TAP M&E), which has its facilities in Humberto Delgado Airport, Lisbon, Portugal. It includes 3 hangars that can simultaneously hold 3 WB (wide-body aircraft) and 5 NB (narrow-body aircraft), and it has a total workforce of around 2,000 people. Besides being responsible for the maintenance of the TAP Air Portugal fleet, TAP M&E also provides services for other airline companies. TAP Group also has a company called TAP M&E Brazil that is a separate company from TAP M&E, but both deliver services to TAP fleet. TAP M&E Brazil has two maintenance facilities in Brazil: one at Galeão International Airport, in Rio de Janeiro, and the other at Salgado Filho International Airport, in Porto Alegre (Source: TAP company)

1.2 Problem Definition and Methodology

Optimizing the schedule of the maintenance activities is crucial for airline companies. This research problem consists in scheduling the maintenance checks that need to be performed, in a way that it minimizes the overall cost of the maintenance procedure, and thus reducing maintenance activities and increasing aircraft availability in the process, but without compromising its feasibility.

The main objective of this dissertation is to develop a decision model that schedules all the aircraft maintenance checks that need to be performed and apply it to the case study of the TAP Air Portugal's fleet. The decision model should be able to find an optimized maintenance schedule for a 2-year horizon. This dissertation work follows the efforts from previous dissertations (Martinho (2018) and Fernandes (2019)), which serve as a starting point to the development of the model and heuristic algorithm for the aircraft maintenance scheduling. The model needs to consider several constraints and variables associated with the case study and find an optimized and feasible solution. Furthermore, the model also must be competitive in computational time.

To achieve the proposed goal, the following methodology was set:

- A literature review on maintenance scheduling, namely in transportation systems, using SCOPUS database;
- Extension and further development of a decision model based on a previous optimization model Martinho (2018);
- Development of a heuristic approach to solve approximately the optimization problem and reduce computational time, based on the work of Fernandes (2019);
- Model verification and implementation using a mixed-integer linear programming formulation through illustrative case studies;
- Application to TAP case study, with discussion and analysis of results;
- Conclusions, limitations and future work.

1.3 Outline

The present document is organized in six sections:

1. Introduction - In this first section, a brief contextualization, and an introduction to the research topic is presented. Firstly, it is approached the topic of air mobility both in Europe and Portugal, and then the maintenance checks in aircrafts are presented. Secondly, the problem and objective for this dissertation and the methodology are described. Finally, the document's structure is given.

2. State of the Art - In this section, several articles on the research topic of this dissertation are presented and summarized, to show what has been done and what is the relevance/innovation of the present work. To begin, some articles about maintenance planning in other means of transportation are presented. Furthermore, some articles on the aircraft maintenance planning and checks are referred and summarized. Finally, the gaps and opportunities, and the contributions of the reviewed articles are given.

3. A Mixed-Integer Linear Programming Model - This section describes a new mathematical programming model and formulation for the aircraft maintenance scheduling/planning problem. All the indexes, sets, parameters, decision variables, objective function and constraints associated with the model are provided and discussed. Finally, a heuristic approach is presented.

4. Application – Model Implementation and Verification - In this section the model that was presented before, is now implemented in an industrial solver software called FICO Xpress and applied to an illustrative example to verify it and to ensure its feasibility. Firstly, the implementation on FICO Xpress is explained. Then all the parameters are now associated with values for the illustrative example, which allows to better understand the model formulation. Finally, the results are given and a comparison between both approaches is made.

5. Case Study – TAP Air Portugal, Results and Discussion – This section, firstly, introduces and describes the case study of TAP Air Portugal that is analysed in this dissertation. Afterwards, all the respective specifications and parameters are set and explained in detail, to give a better and broader understanding of the problem. Furthermore, the results for the case study are presented and discussed. The results from both approaches (the exact method using the branch-and-bound approach and the heuristic approach) are shown. Then, the results are analysed and compared in order to take conclusions of the pros and cons of both approaches. Finally, several tests on the model formulation and parameter values are conducted, and consequently a sensitivity analysis study is performed.

6. Conclusion – In the final section, main conclusions are emphasized, as well as limitations. Further research is also discussed.

2 State of the Art

In this section, several articles on the research topic of this dissertation are presented and summarized, to provide the research background and to show the relevance/innovation of the present work. Firstly, some research works on maintenance planning in other means of transportation are presented. Then, some articles on the aircraft maintenance planning and checks are referred and summarized. Finally, the research gaps and opportunities are identified, and the contributions of the reviewed articles are given.

2.1 Maintenance planning in other means of transportation

Haghani and Shafahi (2002) addresses the problem of scheduling bus maintenance activities. The main objective of this article is to reduce the unavailability of buses, by taking into account both the daily operating schedule and the available maintenance resources, so that the maintenance activities occur during bus idle time. For this, three different integer linear programming approaches are used, and four heuristic algorithms are tested, which reflects on a significant reduction of the computational time in comparison to the integer linear programming approaches, while still achieving results remarkably close to the optimal solution. The model outputs a schedule of all the maintenance activities for each bus, plus the minimum number of maintenance lines needed for each type of activity.

Adonyi *et al.* (2013) develops a P-graph methodology to address the problem of bus maintenance planning. This model considers not only both the daytime and night time, but also the different maintenance operations, materials needed and operating units, while guaranteeing that there are enough buses available for each period. It also shows that the P-graph methodology is versatile and can be applied to other areas.

Fernandes (2019) deals with the minimization of bus preventive maintenance total costs applied to a Portuguese bus operating company. To achieve this, the following three different methods are applied: an extension and improvement of a mixed integer linear programming (MILP) model from a previous work (Martins 2018); a parallel solving approach; and a heuristic approach. All the models output a technical planning schedule for the bus maintenance operations. The heuristic approach, even though it solves the problem approximately and thus the optimal solution cannot be guaranteed, it achieves a particularly good result for the cost minimization and for the computational time. In fact, the heuristic approach followed in Fernandes (2019) serves as a basis for the heuristic algorithm developed in the present dissertation.

Méchain *et al.* (2020) develops a mixed integer linear programming (MILP) model to solve the problem of preventive maintenance planning for a Portuguese train operating company. The main objective is to minimize the overall cost of the maintenance planning, by considering the maintenance facilities capacity and technical costs associated with the maintenance actions, shunting, spare parts, and penalties for early maintenance. The optimization model sets all the different maintenance actions, the maintenance yard lines, and which spare parts are needed for each train unit at each week of the planning horizon.

Martins *et al.* (2021) focuses on the problem of scheduling maintenance tasks and crew in a bus operating company. The proposed methodology uses an integer linear programming (ILP) model that minimizes the costs related to the maintenance actions and bus unavailability, followed by a heuristic approach that solves the problem individually and sequentially for each bus, thus reducing computational time. The model takes into consideration constraints of maintenance resources capacity, such as the depot space availability and the number of maintenance workers. The model is applied to a case study of a Portuguese bus operating company and outputs an optimized maintenance schedule that includes what type of maintenance worker is assigned to each bus and at what day and time.

Alves and Andrade (2021) study the problem of daily scheduling the maintenance technicians in a railway depot for a Portuguese train operating company. A MILP model is formulated in order to minimize labour force costs, while considering the different skills required for each maintenance task and the qualified technicians to perform such tasks. Moreover, the model considers not only a feasible maintenance plan for the company´s fleet, but also the rolling stock schedule, which creates a decision framework to achieve an optimized daily maintenance crew schedule.

2.2 Aircraft maintenance planning and checks

Siriam and Haghani (2003) addresses the problem of cost minimization on scheduling aircraft maintenance activities. To solve this problem, a MILP model is provided and a heuristic approach is presented, which solves the problem approximately in a reasonable lower computational time, without compromising its feasibility, mainly for large sized problems. From a given flight schedule, the main objective is to schedule aircraft maintenance of type A and B, during flight inactivity (late evening/early morning), while taking into account, not only constraints such as maintenance costs, fleet characteristics or the maintenance facilities in each different city, but also cost penalties on the re-assignment of aircrafts from the flight schedule.

Beliën *et al.* (2013) deals with the scheduling of the workforce, for an aircraft maintenance company. In order to schedule the personnel needed for the maintenance lines, and also to define a staffing decision, a MILP model was formulated and an enumerative MILP algorithm with a bounding approach was studied for a case study of a major airline maintenance company servicing at Brussels airport. The problem demonstrated that the enumeration scheme is quite effective, comparing to the normal MILP approach.

Van den Bergh *et al.* (2013) conducts an extensive and thoroughly research on aircraft maintenance operations and its correlation with other airline operations. Several technical aspects were analysed, such as types of maintenance, (integrated) airline scheduling, maintenance workforce, personnel training, facilities location, and maintenance optimization.

Bazargan (2015) presents an optimization mathematical model to define an aircraft dispatching strategy, in order to minimize the maintenance cost and increase aircraft availability. A case study was conducted on a flight school, where the strategy at place was dispatching to training sessions, the aircraft that was closest to its scheduled maintenance, in terms of tach times (i.e., the time the engine is running). By means of a MILP model, the optimization showed results in the order of 2% - 4.6% in cost reductions and an improvement in aircraft availability. Bazargan (2015) also stated that this model could be adapted to rental cars or trucking companies.

Garvranis and Kozanidis (2015) develops a mixed integer programming model (MIP) to solve the flight and maintenance planning problem for the Hellenic Air Force. To determine which aircrafts fly or stay grounded for maintenance activities, for a multi-period planning horizon, the exact solution algorithm takes into account the maintenance requirements and aims to maximize aircraft availability. For reasonable computational times, this model can present an optimal solution for large realistic problems.

Saltoğlu *et al.* (2016) studies the problem of aircraft maintenance and the inherent direct and indirect costs. Contrary to previously articles, which only consider direct maintenance costs such as workforce or material costs, this research proposes an innovative model that calculates the indirect maintenance cost of aircraft downtime during the time that maintenance actions are performed, by also taking into account the influence of season characteristics. The main objective of this model is to serve as a decision support system for operators to determine the best time to schedule maintenance and reduce maintenance operating costs. Saltoğlu *et al.* (2016) not only described the different types of maintenance checks, which are the Line, A, B, C, and D types, but also the different threshold values of maintenance checks intervals for each type of maintenance, and for each type of aircraft.

Qin *et al.* (2018) focuses on the problem of scheduling aircraft hangar maintenance from the perspective of an outsourced maintenance service company, which consists of scheduling the maintenance requests, while considering the hangar parking layout plans and minimizing penalty costs. Therefore, a MILP mathematical model is formulated, considering as constraints the maintenance requests from the airline companies, the parking availability on the maintenance hangars and the roll in and roll out paths of the aircrafts to avoid blockage. Furthermore, an event-based discrete time MILP model to increase efficiency, and a rolling horizon approach for large instances and longer periods, are tested and analysed.

Martinho (2018) deals with the minimization of the total costs associated with aircraft preventive maintenance applied to a Portuguese airline company, which is the same company explored in this dissertation (TAP Air Portugal). Therefore, a MILP model was developed to optimize the costs associated with maintenance checks and associated with aircraft downtime. The checks studied are grouped in different check types A (short-term) and check types C (long-term), because of airline company inputs for the research. The model considers constraints on threshold values for flight hours, flight cycles and days for maintenance checks to occur and constraints regarding hangar capacity availability throughout the planning horizon. Finally, the model outputs a maintenance schedule for the airline company fleet of 45 aircrafts during a 6-month period, which showed promising results in comparison to the original maintenance schedule.

Deng *et al.* (2019) presents a practical dynamic programming-based methodology for the problem of aircraft maintenance checks scheduling, during a long-term planning horizon. The model uses a forward induction approach, plus a thrifty algorithm to estimate future consequences of a scheduled action. The main objective is to minimize the total flight hours wasted between A-type and C-type checks intervals, thus increasing aircraft availability and reducing costs, while taking into account aircraft safety requirements. A case study of an A320 family fleet from a European airline is analysed, which showed promising results regarding the computational time and cost reductions.

Since operators usually create groups of maintenance tasks and perform them at a check event, for the Airbus 320 family, the typical events are the A-check (light check) and the C-check (base check). Although aircraft utilization and operators' rules determine the task packaging, Airbus sets the intervals between same type checks, by using not only the operational units of flight hours and flight cycles, but also the calendar units of months. The values set for the A-type check are 750 flight hours, 750 flight cycles, and 4 months, while for the C-type check are 8000 flight hours and 24 months. Consider that these values are for the Airbus 320 family, which besides the A320 itself, also includes the A318, A319 and A321 (Airbus, 2019). It is important to note that the different airlines share information between themselves, and that Airbus reviews the thresholds intervals from time to time, by analysing failures and data from airline companies. This centralization of operations allows for cooperation and information sharing.

2.3 Research Gaps and Opportunities

After reviewing the articles in the previous sections, it is clear that research on maintenance planning in aircrafts or other means of transport has come a long way, and each article helped to better understand the concepts of maintenance planning, aircraft maintenance operations and regulations. Nonetheless, improvements can still be studied and explored. A few of these researches serve as a basis for this present dissertation, such as the work carried out by Martinho (2018), as it schedules the necessary maintenance checks required by TAP M&E, but not only does it for a 6-month period, which is considered too short for the aviation industry usual planning horizon, but also does not take into account the influence of the variation in seasonality on the airline companies' businesses. Besides, the model does not guarantee finding an optimal solution, achieving a solution with an approximately 9% of optimality gap in 24 hours of computational time, which is considered high for this type of problem. Other articles that serve as a basis for the current research, are a master dissertation by Fernandes (2019) and the research paper (Martins *et al.* 2021), which introduce a heuristic to approximately solve the optimization problem and present promising results, but for a bus operating company, which is a form of transport that does not have the same strict maintenance requirements as aircrafts.

Therefore, an adapted MILP model was formulated, which includes all the requirements that need to be considered for the aircraft maintenance scheduling problem of TAP Air Portugal, and a heuristic algorithm was developed and implemented to solve the model, as it will be explored in the following subsections.

2.4 Contributions of the Research

The contributions of each article previously reviewed are presented and summarized in the next table 2.1.

Table 2.1 - Summary of the analysis of the articles on maintenance scheduling and its contributions.

Reference	Year	Keywords	Technique(s)	Contribution
Haghani and Shafahi	2002	Bus Transit, Maintenance, Scheduling, Optimization model, Heuristic algorithm	MILP	Minimize operating buses unavailability and maximize efficiency of maintenance facilities
Siriam and Haghani	2003	Aircraft, Maintenance, Scheduling, Optimization, Heuristic	MILP, Heuristic	Minimize maintenance checks costs and penalty costs of re-scheduling flights considering a previous flight schedule
Beliën *et al.*	2013	Staffing, Scheduling, Aircraft line maintenance, Mixed integer programming	MILP, Enumerative algorithm	Schedule workforce for aircraft maintenance by considering both a staffing and scheduling decision
Bazargan	2015	Mathematical model, Optimization, Integer programming, Dispatching	MILP	Minimize costs and increase aircraft availability by defining a dispatching strategy
Garvranis and Kozanidis	2015	Mixed Integer Programming, Fleet Availability, Flight and Maintenance Planning, Exact Solution Algorithm	MIP	Maximize aircraft fleet availability for considerably large realistic problems
Martinho	2018	Maintenance Planning, Air Transportation, Optimization, Scheduling, Mixed Integer Linear Programming	MILP	Minimize maintenance costs and aircraft unavailability
Qin *et al.*	2018	Aircraft maintenance scheduling, Hangar parking layout planning, Mixed Integer Linear Programming, Event-based model, Rolling horizon approach	MILP, Event-based, Rolling horizon	Event-based discrete time MILP model and rolling horizon approach for parking planning and minimizing penalty costs for an aircraft maintenance schedule
Fernandes	2019	Optimization, Maintenance Scheduling, Bus Transport, Mixed Integer Linear Programming, Parallel Solving	MILP, Parallel Solving, Heuristic	Minimize cost in bus preventive maintenance operations and scheduling of maintenance actions and maintenance teams

Deng *et al.*	2019	Scheduling, Aircraft Maintenance, Dynamic Programming, Forward Induction	Dynamic Programming, Forward Induction	Minimize the total flight hours wasted between checks intervals. Reduction in computational time
Méchain *et al.*	2020	Maintenance optimization, Train operating companies, Mixed Integer Linear Programming	MILP	Minimize preventive maintenance costs considering yard capacity and technical constraints
Martins *et al.*	2021	Optimization, Maintenance Scheduling, Integer Linear Programming	ILP, Heuristic	Minimize maintenance costs associated with maintenance tasks and crew and costs associated with bus unavailability
Alves and Andrade	2021	Railway maintenance, Maintenance crew scheduling, Mixed Integer Linear Programming, Maintenance planning	MILP	Schedule maintenance crew, by considering different skilled workforce, while minimizing associated labour costs

3 A Mixed-Integer Linear Programming Model

This section describes a new mathematical programming model and formulation for the aircraft maintenance scheduling/planning problem. All the indexes, sets, parameters, decision variables, objective function and constraints associated with the model are provided and discussed. Finally, a heuristic approach to solve the mathematical model is also presented.

3.1 Model Background and Improvements

The present mathematical model is an improvement of the research work by Martinho (2018). The previous model version had a time horizon of only 6 months, with a time step of 1 day. Moreover, the heuristic algorithm is developed based on a previous research carried out by Fernandes (2019).

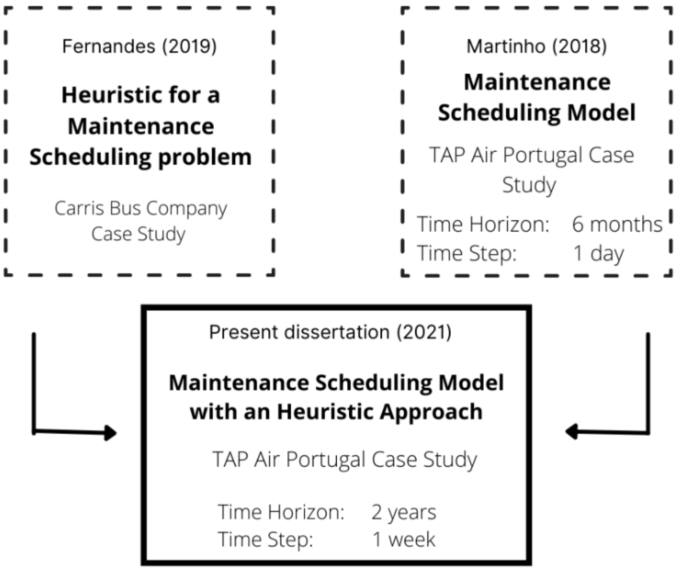


Figure 3.1 – Relation of the present dissertation with previous works.

For the aircraft maintenance planning, a time horizon of just 6 months might be considered too short to provide a reasonable plan for the maintenance activities as it might lose medium-term effects in certain maintenance tasks. Therefore, the model was extended to include a time horizon of 2 years. The change of the time step, from 1 day to 1 week, aims to reduce the computational time, which otherwise would be too long if the 2 years would be scheduled day by day. Besides, the previous model did not consider penalties on checks done during high season and that for check C the aircraft has an unavailability of two weeks. Furthermore, to reduce computational time, without compromising the solution feasibility and optimization, a heuristic approach was implemented. This heuristic approach is done mainly because in the previous model by Martinho (2018), even though it achieved a good solution in terms of optimization values, the computational time was still too high for a problem of this complexity. In order to implement a heuristic procedure, several articles were studied, and the heuristic algorithm was adapted from Fernandes (2019) and Martins *et al.* (2021), which showed promising results on the heuristic procedure, not only for the computational time, but also for the optimization results.

3.2 Model Formulation

This model formulation tackles the problem of scheduling the maintenance activities for the fleet of an airline company, such as TAP Air Portugal aircraft fleet. The main objective is to reduce the maintenance costs, which include the costs due to maintenance checks, and the unavailability (or penalty) costs of the aircraft being grounded for the maintenance operation to be done. The unavailability cost is identified by TAP Air Portugal and is quantified as the renting of an aircraft to compensate the service, but may also include penalties, loss of revenues and other economic impacts, due to the impossibility of the aircraft to satisfy passengers' demand.

The model must consider several variables and constraints. Firstly, there are the variables and constraints regarding the flight hours, flight cycles and weeks that an aircraft has at each given week. Each aircraft at the beginning of the planning horizon (*t*=0), has a value of flight hours, flight cycles and weeks, since its last A-type and C-type check, which are represented by the constants present on the left of the vertical line of the start of the planning horizon, as shown in figure 3.2. When the planning horizon starts, each aircraft has the weekly flight hours, weekly flight cycles and weeks summed to their previous values, represented by $\boldsymbol{FHw_{p,t}}$ and $\boldsymbol{FCw_{p,t}}$, for the flight hours and flight cycles, respectively. These values are added up each week, until they reach a threshold value that obligates the aircraft to undergo an A-type or C-type check, represented by the max values on Figure 3.2. When the maintenance occurs, the values need to be set to zero, and start summing the weekly values from scratch, until achieving the next threshold value again, and so on, until the planning horizon ends.

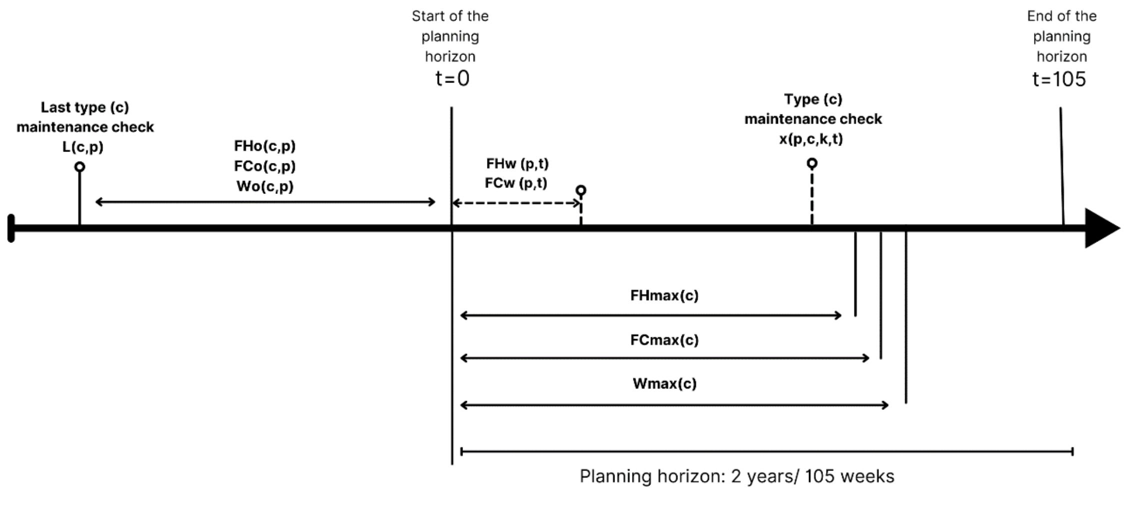


Figure 3.2 - Model scheme [adapted from: (Martinho, 2018)]

Moreover, the model needs to consider the hangar capacity availability for each week during the planning horizon, so that if the hangar for a certain week is full, an aircraft must be scheduled for a different week, even if the values of the flight hours, flight cycles or weeks, do not achieve the threshold values, at that certain week.

Besides, just like other transportation systems that involve passengers, there are times where the demand and associated flows increase. In the aviation industry, this time usually comes during holidays in the summer or during Christmas, known as the high season, which means that the model needs to include such details. To address this problem, the model ensures that a penalty cost is given, if a C-type maintenance check is done during the high season, since it will bring a higher loss of revenues for the airline company due to aircraft unavailability. The penalty is given only to C-type maintenance checks, since they take up to 2 weeks to be executed, while the A-type maintenance checks usually take 1 to 2 days to be done and are impossible to avoid during such a large period of time like summer, since they have lower threshold values in comparison to the C-type maintenance check.

With this brief contextualization and presentation of the model, a complete explanation and definition of the model and its formulation is provided in the next sections.

3.2.1 Indexes

$\boldsymbol{p}$	plane
$\boldsymbol{c}$	type of maintenance check
$\boldsymbol{t}$	time period (week)
$\boldsymbol{k}$	number of maintenance check

3.2.2 Constants

$\boldsymbol{Np}$	number of planes
$\boldsymbol{Nc}$	number of different types of maintenance checks (Two check types: A-type and C-type, $Nc = 2$)
$\boldsymbol{Nw}$	number of weeks in planning horizon
$\boldsymbol{Nk_c}$	number of maintenance checks of type *c*
$\boldsymbol{M}$	large number
$\boldsymbol{\varepsilon}$	small number

3.2.3 Sets

$\boldsymbol{P}$	set of planes *p*
$\boldsymbol{C}$	set of types of maintenance checks *c*
$\boldsymbol{T}$	set of time periods (weeks) *t*
$\boldsymbol{T^{HS}}$	set of time periods (weeks) of high season *t*

K_c set of number of maintenance checks *k* of type *c*

3.2.4 Parameters

S_t available hangar maintenance slots, in week *t*

$L_{c,p}$ last maintenance check number of type c, for plane *p*

$FH^0_{c,p}$ accumulated Flight Hours (FH), since last maintenance check of type *c*, for plane *p*, in week *t=0* (i.e., at the beginning of the planning horizon)

$FC^0_{c,p}$ accumulated Flight Cycles (FC), since last maintenance check of type *c*, for plane *p*, in week *t=0* (i.e., at the beginning of the planning horizon)

$W^0_{c,p}$ accumulated Weeks (W), since last maintenance check of type *c*, for plane *p*, in week *t=0* (i.e., at the beginning of the planning horizon)

$FHw_{p,t}$ estimated weekly Flight Hours (FH), for plane *p*, in week *t*

$FCw_{p,t}$ estimated weekly Flight Cycles (FC), for plane *p*, in week *t*

FH^{max}_c threshold values for maximum Flight Hours (FH), between two consecutive maintenance checks of type c

FC^{max}_c threshold values for maximum Flight Cycles (FC), between two consecutive maintenance checks of type c

W^{max}_c threshold values for maximum Weeks (W), between two consecutive maintenance checks of type c

γ_1 , γ_2 decision weights for the objective function

c_{un} unavailability cost

c_c cost of maintenance check of type *c*

3.2.5 Decision Variables

This section identifies the decision variables in the model formulation.

$$x_{p,c,k,t} = \begin{cases} 1 \text{ if maintenance activity type } c\text{, number } k\text{, is performed, on plane } p\text{, on week } t \\ 0 \text{ otherwise} \end{cases}$$

$$y_{p,t} = \begin{cases} 1 \text{ if plane } p\text{, is on hangar, on week } t \\ 0 \text{ otherwise} \end{cases}$$

$FH_{p,c,t}$ accumulated flight hours, for plane *p*, since last type check *c*, on week *t*

$FC_{p,c,t}$ accumulated flight cycles, for plane *p*, since last type check *c*, on week *t*

$W_{p,c,t}$ accumulated weeks, for plane *p*, since last type check *c*, on week *t*

Note: A-type checks corresponds to $c = 1$, and C-type checks correspond to $c = 2$.

3.2.6 Constraints

To achieve the objective function, first it is necessary to consider the constraints of the problem, not only to reach the best solution, but also to be a feasible and realistic one.

The equations of the constraints used in this model are presented, followed by a detailed explanation of each one.

$$x_{p,c,k,t} \in \{0,1\} \qquad \forall\, p \in P, c \in C, k \in K_c, t \in T \qquad (1)$$

$$y_{p,t} \in \{0,1\} \qquad \forall\, p \in P, t \in T \qquad (2)$$

$$FH_{p,c,t} \leq FH_c^{max} \qquad \forall\, p \in P, c \in C, t \in T \qquad (3)$$

$$FC_{p,c,t} \leq FC_c^{max} \qquad \forall\, p \in P, c \in C, t \in T \qquad (4)$$

$$W_{p,c,t} \leq W_c^{max} \qquad \forall\, p \in P, c \in C, t \in T \qquad (5)$$

$$FH_{p,c,t} = FH_{c,p}^{0} \qquad \forall\, p \in P, c \in C, t \in \{1\} \qquad (6)$$

$$FH_{p,c,t} \geq FH_{p,c,t-1} + FHw_{p,t} \times \left(1 - \sum_{k \in K_c} x_{p,c,k,t}\right) - FH_c^{max} \times \sum_{k \in K_c} x_{p,c,k,t} \qquad \forall\, p \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (7)$$

$$FH_{p,c,t} \geq \varepsilon \times \sum_{k \in K_c} x_{p,c,k,t} \qquad \forall\, p \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (8)$$

$$FC_{p,c,t} = FC_{c,p}^{0} \qquad \forall\, p \in P, c \in C, t \in \{1\} \qquad (9)$$

$$FC_{p,c,t} \geq FC_{p,c,t-1} + FCw_{p,t} \times \left(1 - \sum_{k \in K_c} x_{p,c,k,t}\right) - FC_c^{max} \times \sum_{k \in K_c} x_{p,c,k,t} \qquad \forall\, p \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (10)$$

$$FC_{p,c,t} \geq \varepsilon \times \sum_{k \in K_c} x_{p,c,k,t} \qquad \forall\, p \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (11)$$

$$W_{p,c,t} = W_{c,p}^{0} \qquad \forall\, p \in P, c \in C, t \in \{1\} \qquad (12)$$

$$W_{p,c,t} \geq W_{p,c,t-1} + 1 \times \left(1 - \sum_{k \in K_c} x_{p,c,k,t}\right) - W_c^{max} \times \sum_{k \in K_c} x_{p,c,k,t} \qquad \forall\, p \in P, c \in C, t \in \{2, \dots, Nw\} \qquad \mathbf{(13)}$$

$$W_{p,c,t} \geq \varepsilon \times \sum_{k \in K_c} x_{p,c,k,t} \qquad \forall\, p \in P, c \in C, t \in \{2, \dots, Nw\} \qquad \mathbf{(14)}$$

$$\sum_{c \in C} \sum_{k \in K_c} x_{p,c,k,t} \leq M \times y_{p,t} \qquad \forall\, p \in P, t \in T \qquad \mathbf{(15)}$$

$$\sum_{k \in K_c} x_{p,c,k,t} \leq y_{p,t+1} \qquad \forall\, p \in P, c \in \{2\}, t \in \{1, \dots, 104\} \qquad \mathbf{(16)}$$

$$\sum_{k \in K_c} x_{p,c,k,t} + \sum_{k \in K_c} x_{p,c,k,t+1} \leq 1 \qquad \forall\, p \in P, c \in \{1\}, t \in \{1, \dots, 104\} \qquad \mathbf{(17)}$$

$$\sum_{p \in P} y_{p,t} \leq s_t \qquad \forall\, t \in T \qquad \mathbf{(18)}$$

$$\sum_{k \in K_c} x_{p,c,k,t} \leq 1 \qquad \forall\, p \in P, c \in \{1\}, t \in T \qquad \mathbf{(19)}$$

$$\sum_{k \in K_c} x_{p,c,k,t} \leq 1 \qquad \forall\, p \in P, c \in \{2\}, t \in T \qquad \mathbf{(20)}$$

$$\sum_{t \in T} \sum_{k \in \{2,\dots,4\} | k = n+1} x_{p,c,k,t} \geq 1 \qquad \forall\, p \in P, c \in \{1\}, n \in \{1,2,3\} \mid L_{c,p} = n \qquad \mathbf{(21)}$$

$$\sum_{t \in T} x_{p,c,1,t} \geq 1 \qquad \forall\, p \in P, c \in \{1\} \mid L_{c,p} = 4 \qquad \mathbf{(22)}$$

$$\sum_{t \in T} \sum_{k \in \{2,\dots,12\} | k = n+1} x_{p,c,k,t} \geq 1 \qquad \forall\, p \in P, c \in \{2\}, n \in \{1, \dots, 11\} \mid L_{c,p} = n \qquad \mathbf{(23)}$$

$$\sum_{t \in T} x_{p,c,1,t} \geq 1 \qquad \forall\, p \in P, c \in \{2\} \mid L_{c,p} = 12 \qquad \mathbf{(24)}$$

$$\sum_{t' \in T | t' \leq t} x_{p,c,k+1,t'} \leq \delta_{k,\mathrm{L_{c,p}}} + \sum_{t'' \in T | t'' \leq t} x_{p,c,k,t''} \qquad \forall\, p \in P, c \in C, k \in K_c/\{Nk_c\}, t \in T/\{1\} \qquad \mathbf{(25)}$$

$$\sum_{t' \in T | t' \leq t} x_{p,c,1,t'} \leq \delta_{Nk_c,\mathrm{L_{c,p}}} + \sum_{t'' \in T | t'' \leq t} x_{p,c,Nk_c,t''} \qquad \forall\, p \in P, c \in C, t \in T/\{1\} \qquad \mathbf{(26)}$$

Constraints **(1)** and **(2)** define as binary variables, the decision variables $\boldsymbol{x_{p,c,k,t}}$ and $\boldsymbol{y_{p,t}}$, respectively.

Constraints **(3)**-**(5)** are responsible to guarantee that the threshold values between A-types and C-types maintenance checks are not exceeded for any plane $\boldsymbol{p}$ at any time $\boldsymbol{t}$. Constraint **(3)** is for the flight hours threshold values, **(4)** is for the flight cycles threshold values, and **(5)** is for the week's threshold values, for each type of maintenance check.

Before continuing the constraints explanation, consider that for the set $\boldsymbol{K_1}$, which represents the set for the A-type maintenance checks, the values are {1,2,3,4}, while for the set $\boldsymbol{K_2}$, which represents the set for the C-type maintenance checks, the values are {1, …,12}. This means that there are four different numbers of A-type checks, while there are twelve different numbers of C-type checks.

Constraints **(6)-(14)** are responsible to ensure that a maintenance check occurs when the accumulation of flight hours, flight cycles or weeks reaches the threshold values and that in each week, the weekly values of these parameters are added. These constraints can be divided into three groups of three to be better explained. The first group **(6)-(8)** of constraints regards to flight hours for the maintenance check type $\boldsymbol{c}$. Constraint **(6)** sets the value of the decision variable $\boldsymbol{FH_{p,c,t}}$ equal to the value of the parameter $\boldsymbol{FH_{c,p}^{0}}$, which is the value of accumulated flight hours for aircraft $\boldsymbol{p}$ at the beginning of the time horizon $(t = 1)$, since last maintenance check type $\boldsymbol{c}$ was done. Constraint **(7)** ensures the continuous accumulation of flight hours $\boldsymbol{FH_{p,c,t}}$ throughout the time horizon, by adding the weekly flight hours $\boldsymbol{FHw_{p,t}}$ to the previous week value $\boldsymbol{FH_{p,c,t-1}}$, while checking if the value of $\boldsymbol{FH_{p,c,t}}$ remains under the threshold value of flight hours $\boldsymbol{FH_{c}^{max}}$ for maintenance check type $\boldsymbol{c}$. If for a certain week $\boldsymbol{t}$, the value of $\boldsymbol{FH_{p,c,t}}$ is going to be higher than the $\boldsymbol{FH_{c}^{max}}$, then a maintenance check type $\boldsymbol{c}$ needs to occur in that week $\boldsymbol{t}$ and the decision variable $\boldsymbol{x_{p,c,k,t}}$ will be equal to one. Constraint **(8)** guarantees that if a maintenance check type $\boldsymbol{c}$ needs to occur in a certain week $\boldsymbol{t}$, then the value of $\boldsymbol{FH_{p,c,t}}$ is set to zero for that same week $\boldsymbol{t}$. For the remaining two groups of three, the formulation has the same purpose and similarity, but for different types of counters. Constraints **(9)-(11)** are for flight cycles for maintenance checks type $\boldsymbol{c}$, while constraints **(12)-(14)** are for weeks for maintenance checks type $\boldsymbol{c}$.

Constraint **(15)** ensures that whenever a plane $\boldsymbol{p}$ needs to undergo a maintenance check at a certain week $\boldsymbol{t}$, defined by the decision variable $\boldsymbol{x_{p,c,k,t}}$, then the aircraft must go to the hangar at the same week $\boldsymbol{t}$, defined by the decision variable $\boldsymbol{y_{p,t}}$. Constraint **(16)** defines that for C-type maintenance checks, the aircraft needs to stay at the hangar one more week, adding up to two weeks of downtime, while constraint **(17)** states that two different A-type maintenance checks cannot be done in two sequential weeks, to avoid the programme solving to assign two consecutive A-type maintenance checks to be done only because the aircraft is already at the hangar for the C-type maintenance check. Constraint **(18)** guarantees that the sum of the aircrafts at the hangar at a certain week $\boldsymbol{t}$, does not surpasses the hangar capacity of maintenance slots available $\boldsymbol{s_t}$, for that week $\boldsymbol{t}$.

Constraints **(19)** and **(20)** state that a plane $\boldsymbol{p}$ cannot have two A-type maintenance checks or two C-type maintenance checks, respectively, in the same week $\boldsymbol{t}$.

Constraints **(21)-(26)** ensure that the cycle order of the different numbers of A-type and C-type maintenance checks are respected, as shown in figure 3.3.

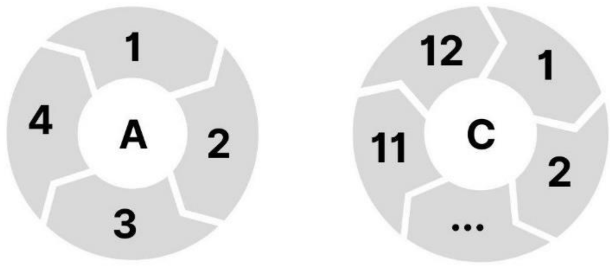


Figure 3.3 - Cycle of numbers of A-type and C-type maintenance checks

Constraints **(21)-(24)** guarantee that for a certain number of last maintenance check before the start of the planning horizon $\boldsymbol{L_{c,p}}$, the next number of maintenance check that needs to occur follows the cycle order previously showed. Constraints **(21)** and **(22)** refer to A-type maintenance checks, while constraints **(23)** and **(24)** refer to C-type maintenance checks.

Constraints **(25)** and **(26)**, ensure that the cycle order is respected throughout the entire planning horizon. In both equations, the δ represents a Kronecker delta, i.e., for $\delta_{k,\mathrm{L_{c,p}}}$, δ is only equal to 1 if $k = L_{c,p}$, or else the δ is equal to zero. To better explain how both constraints work, Figure 3.4 will give an overall view of what it would like for a plane $\boldsymbol{p}$, that had a last maintenance check A1, before the start of the planning horizon.

$$\textstyle\sum_{t1 \in T | t1 \leq t} x_{p,1,3,t1} \leq \sum_{t0 \in T | t0 \leq t} x_{p,1,2,t0}, \quad \forall\, p \in P,\ t \in T/\{1\} |\ L_{1,p} = 1$$

$$\textstyle\sum_{t2 \in T | t2 \leq t} x_{p,1,4,t2} \leq \sum_{t1 \in T | t1 \leq t} x_{p,1,3,t1}, \quad \forall\, p \in P,\ t \in T/\{1\} |\ L_{1,p} = 1$$

$$\textstyle\sum_{t3 \in T | t3 \leq t} x_{p,1,1,t3} \leq \sum_{t2 \in T | t2 \leq t} x_{p,1,4,t2}, \quad \forall\, p \in P,\ t \in T/\{1\} |\ L_{1,p} = 1$$

$$\textstyle\sum_{t4 \in T | t4 \leq t} x_{p,1,2,t4} \leq 1 + \sum_{t3 \in T | t3 \leq t} x_{p,1,1,t3}, \quad \forall\, p \in P,\ t \in T/\{1\} |\ L_{1,p} = 1$$

Figure 3.4 - Example of how constraints (26) and (27) work

3.2.7 Objective function

The objective of this model, as stated before, is to minimize the cost associated with maintenance activities and unavailability of the aircraft, while taking into account the previous constraints. Considering this, the following objective function was created and consequently minimized.

$$\text{minimize} \left(\sum_{p \in P} \sum_{t \in T} c_{un} \times y_{p,t} + \left(\sum_{p \in P} \sum_{t \in T} \sum_{k \in K_c} c_1 \times x_{p,1,k,t} + c_2 \times x_{p,2,k,t} \right) + \gamma 1 \times \left(\sum_{p \in P} \sum_{c \in C} \sum_{t \in T} t \times FH_{p,c,t} \right) + \gamma 2 \times \left(\sum_{p \in P} \sum_{k \in K_c} \sum_{t \in T^{HS}} c_2 \times x_{p,2,k,t} \right) \right) \quad \mathbf{(27)}$$

The first and second terms on the objective function are the main objectives and are responsible for the real overall cost value of the maintenance activities, according to the schedule that will be defined. The first term considers the unavailability cost whenever an aircraft is in the hangar for a maintenance activity, while the second term considers the costs of the actual maintenance checks, both A and C, each time one is scheduled to be done.

The third term ensures that the maintenance activities are done as close to the threshold values as possible at the start of the scheduling horizon, when there are less factors that could interfere with the normal realization of the schedule activities, such as delays on the maintenance procedures. Then at the end of the schedule, the maintenance activities are done earlier than the threshold values, so there is time for any changes in schedule that may need to be done, due to previous delays.

The fourth term is responsible to give an extra penalty cost for when maintenance check type C is done on the high season. The high season is considered on summertime and on Christmas and new years' time, since it´s when the flux of passengers is higher, and the realization of the check C in this time can bring extra unavailability costs for the airline companies.

The objective function value will be the sum of all these terms with different decision weights, accordingly to the values associated with $\gamma 1$ and $\gamma 2$, that will be further defined and tested. Minimizing this value, will result in the best solution for the model. Note that the value for best solution will be the overall value of the objective function, but the real cost value of the schedule maintenance checks is only the sum of the first and second terms.

3.2.8 Post-Processing Data

To reach the exact values for flight hours, flight cycles and weeks that the planes have between checks A-type and between checks C-type, along the planning horizon, a post-processing is created, since the decision variables $\boldsymbol{FH_{p,c,t}}$, $\boldsymbol{FC_{p,c,t}}$ and $\boldsymbol{W_{p,c,t}}$ give approximated values of flight hours, flight cycles and weeks, respectively, and only serve as a decision value.

Firstly, the parameters for these values are created.

$\boldsymbol{TFH_{c,p,t}}$	total accumulated flight hours, since last maintenance check of type *c*, for plane *p*, on week *t*
$\boldsymbol{TFC_{c,p,t}}$	total accumulated flight cycles, since last maintenance check of type *c*, for plane *p*, on week *t*
$\boldsymbol{TW_{c,p,t}}$	total accumulated weeks, since last maintenance check of type *c*, for plane *p*, on week *t*

After the value for the objective function is reached, the total flight hours, flight cycles and weeks are calculated for each plane, using the values of the decision variables and the data of some parameters, that were defined before the model computation. The post-processing expressions are created and explained.

$$TFH_{c,p,t} \coloneqq FH^0_{c,p} \qquad \forall\, p \in P, c \in C, t \in \{1\} \qquad (\mathbf{28})$$

$$TFH_{c,p,t} \coloneqq (TFH_{c,p,t-1} + FHw_{p,t}) \times \left(1 - \sum_{k\, \in\, K_c} getsol(x_{p,c,k,t})\right) \qquad \forall\, p\ \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (\mathbf{29})$$

$$TFC_{c,p,t} \coloneqq FC^0_{c,p} \qquad \forall\, p \in P, c \in C, t \in \{1\} \qquad (\mathbf{30})$$

$$TFC_{c,p,t} \coloneqq (TFC_{c,p,t-1} + FCw_{p,t}) \times \left(1 - \sum_{k\, \in\, K_c} getsol(x_{p,c,k,t})\right) \qquad \forall\, p\ \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (\mathbf{31})$$

$$TW_{c,p,t} \coloneqq W^0_{c,p} \qquad \forall\, p \in P, c \in C, t \in \{1\} \qquad (\mathbf{32})$$

$$TW_{c,p,t} \coloneqq (TW_{c,p,t-1} + 1) \times \left(1 - \sum_{k\, \in\, K_c} getsol(x_{p,c,k,t})\right) \qquad \forall\, p\ \in P, c \in C, t \in \{2, \dots, Nw\} \qquad (\mathbf{33})$$

Expressions **(28)** and **(29)** are responsible to create the values for the parameter of total flight hours in between maintenance checks type $\boldsymbol{c}$, for each plane $\boldsymbol{p}$, at each single week $\boldsymbol{t}$. The other expressions have the same purpose, except expressions **(30)** and **(31)** are responsible to create the values for the parameter of total flight cycles, while expressions **(32)** and **(33)** are responsible to create the values for the parameter of total weeks.

3.3 Heuristic

In this section, a heuristic approach is presented and an explanation on how it is implemented in the model is given. The heuristic is based on the work of (Fernandes, 2019).

The main objective of using the heuristic approach is to reduce computational time. To achieve this reduction, instead of solving the entire MILP problem at once, the heuristic approach solves the problem sequentially, one aircraft at a time, saving and gathering the results one aircraft after the other. This translates in solving plane number 1 first, and then saving the solution and removing the maintenance hangar slots for the respective time periods for the next plane, which means that when solving for plane number 2, the slots that are occupied by plane number 1 are no longer available to be a solution possibility. This happens sequentially until the problem is solved for all planes.

To implement the heuristic, some changes need to be done on the model formulation. First it is necessary to create a new parameter called $\boldsymbol{Gp_p}$, that is responsible for defining in which order the aircrafts will be solved, i.e., $\boldsymbol{Gp_1}$ will be the first plane to be solved and so on. This way, the first plane to be solved does not mean that it is the aircraft number 1 of the input data order, represented on the set $\boldsymbol{P}$, since, for example, the value of $\boldsymbol{Gp_1}$ could be equal to 9, which results in aircraft number 9 of the input data order being the first aircraft to be solved. The criteria for deciding the order in which the planes are solved and the actual order, will be explained on section 4 for the illustrative example and on section 5 for the case study.

For the heuristic process, a loop is created using the “repeat” function of the Mosel language, in which a new variable $\boldsymbol{NPlane}$ is created with the value of zero and increased by one in each iteration. The variable $\boldsymbol{NPlane}$ is then used on the parameter $\boldsymbol{Gp_{NPlane}}$, to add a new plane to the new set $\boldsymbol{P1}$, which will be the set of planes in the order defined by the criteria previously decided. At the end of each procedure, the achieved solution is saved on the parameter $\boldsymbol{Prop\_x_{p,c,k,t}}$, by using the post-processing expression **(34)**.

$$Prop\_x_{p,c,k,t} \coloneqq getsol(x_{p,c,k,t}) \qquad \forall\, p \in P1, c \in C, k \in K_c, t \in T \qquad (\mathbf{34})$$

To ensure that the solution achieved of each previous plane is considered for each next one, constraint **(35)** is added to the model formulation. This constraint guarantees that if an aircraft has a solution to undergo maintenance activities at certain times t, then the procedure for the next aircraft will take into account that those maintenance checks already occurred and thus “locking” the time and hangar availability slots.

$$x_{p,c,k,t} = Prop\_x_{p,c,k,t} \qquad \forall\, p \in P1\backslash\{Gp_{NPlane}\}, c \in C\,, k \in K_c, t \in T \qquad (\mathbf{35})$$

The loop runs until the iteration value, reaches the total number of aircrafts of the problem, stage where the loop is terminated, and the final solution is retrieved.

The heuristic also has some post-processing expressions that will help the model to get the best solution by reducing small errors between aircrafts in the repeat process, that may occur during the heuristic procedure, i.e., for $\boldsymbol{x_{p,c,k,t}}$ solution values lower than 0.9, the $\boldsymbol{Prop\_x_{p,c,k,t}}$ solution is saved as zero, while for $\boldsymbol{x_{p,c,k,t}}$ solution values higher than 0.9, the $\boldsymbol{Prop\_x_{p,c,k,t}}$ solution is saved as 1.

The following flowchart displayed on Figure 3.5, serves to better understand the previous explanation of the heuristic algorithm and give a better visualization of the entire procedure.

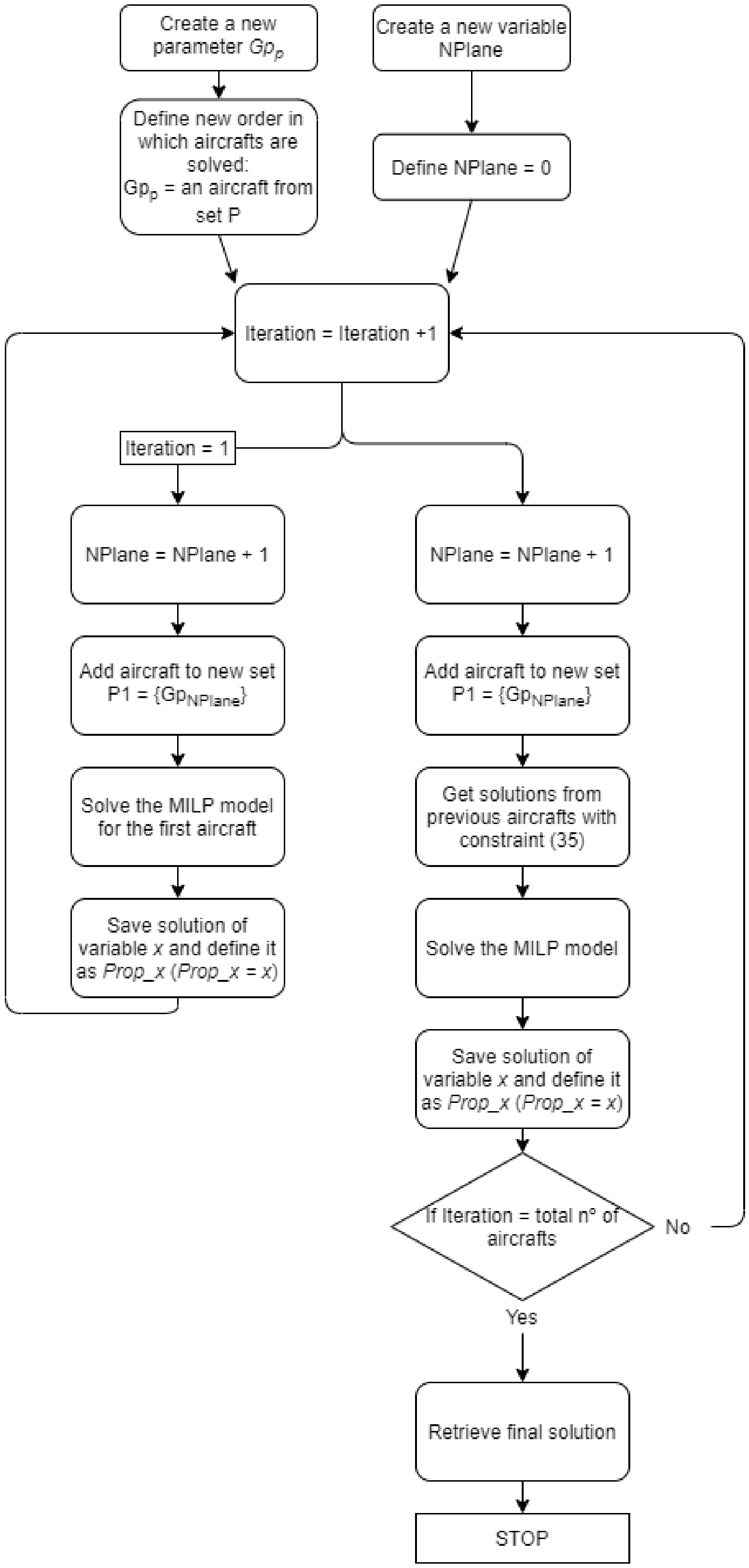


Figure 3.5 - Heuristic algorithm flowchart

4 Application – Model Implementation and Verification

In this section the model presented before is now implemented in a commercial software called FICO Xpress and applied to an illustrative example to verify it and to ensure its feasibility. Firstly, the implementation on FICO Xpress is explained. Then all the parameters are associated with values for the illustrative example, which allows a better understanding of the model formulation. Finally, the results are given and a comparison between both approaches is made.

4.1 Model implementation in FICO Xpress Optimization Software

Nowadays, optimization is regarded as a key factor for companies worldwide, since it finds the best results for any type of problems. “Optimization is the mathematical process of finding the best decision for a given business problem within a defined set of constraints.” (FICO, 2021). The optimization concept is a standard approach for almost all companies when it comes to management of resources, such as employees, equipment or infrastructures, which is the case for the present research, which has to consider the management of the aircrafts’ availability and the reduction of maintenance costs, while taking into account the hangar capacity and aircraft maintenance regulations.

The optimization process is a complex one, and “the importance of properly formulating a design optimization problem must be stressed because the optimum solution will be only as good as the formulation” (Arora, 2012). To tackle such complexity, it not only requires a multi-step approach, but it also requires a continuous search process in order to reach the best possible solution. An overview scheme of the optimization process is displayed in the following Figure 4.1.

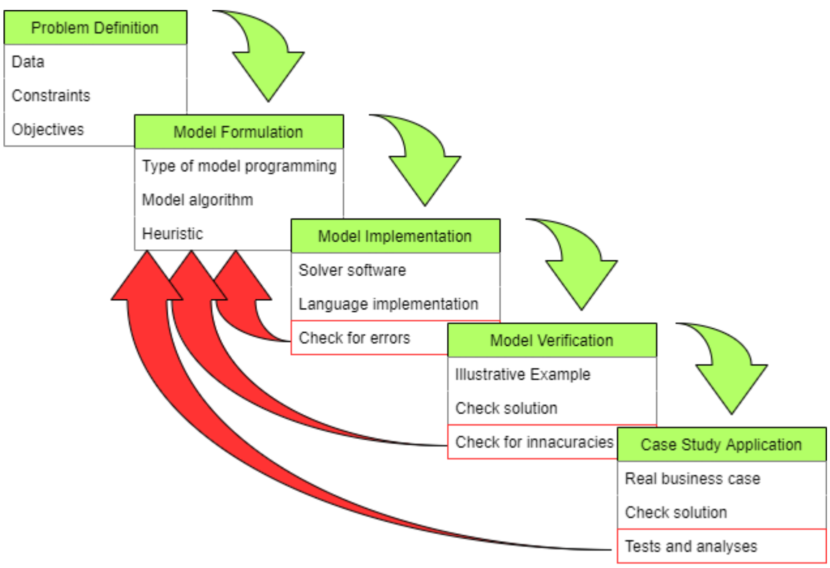


Figure 4.1 - Optimization Process Scheme

To implement the formulated model, there are several options to address this problem, such as MATLAB, CPLEX or FICO Xpress. For the implementation of this model, the software chosen is FICO Xpress Optimization Software. It can solve a wide range of optimization problems, such as linear programming (LP) or mixed-integer linear programming (MILP) (FICO products, 2021). Since the present model formulation has both the objective function and the constraints as linear functions, while having continuous and integer decision variables, the mathematical model is considered to be a Mixed-Integer Linear Programming (MILP) model.

Once the solver was chosen, the mathematical formulation is adapted to the software´s code language, called Mosel. FICO has different modules that need to be selected at the beginning, and one of the modules to be selected is “mmxrps”, which obtains access to the Xpress-Optimizer solver. For the implementation, the model is divided into different sections, by the following order:

1. Declarations
2. Initializations (Input Data)
3. Constraints
4. Objective Function
5. Post-processing
6. Output Data

On the declarations section, the constants, the sets and correspondent indexes, the parameters (arrays of real or integer), and the decision variables (arrays of “mpvar”) are defined. The initializations section is responsible for the input of data necessary to solve the model, which in this case means four different data files. One file that has the information on the accumulated flight hours, flight cycles and weeks since the last A-type and C-type checks, for each aircraft, at the beginning of the planning horizon, and has the information on the hangar capacity, for each week of the planning horizon. Another file that has the information on the number of the last A-type and C-type checks, for each aircraft. The remaining two data files, one is for the weekly flight hours and another one for the weekly flight cycles, for each aircraft, throughout the planning horizon.

The constraints and objective function sections are defined accordingly to the requirements and goals of the company for this research.

Finally, on the output data, besides delivering the decision variables and objective function solutions, some code was added so that, when the model finishes executing, it outputs a data file with the information of when each type and number of maintenance checks occurs, for each aircraft, as the Figure 4.2 shows (“1-2” represents A-type check number 2 and “2-10” represents C-type check number 10). Moreover, it displays a graph at the end, showing how many and which aircrafts are in the hangar per week, as it will be further visualized on Figures 4.8 and 4.13.

```
for the plane #1
At 9 week, a maintenance type 1-2 has to be performed
At 18 week, a maintenance type 2-10 has to be performed
```

Figure 4.2 – Example of output data file with information on scheduled maintenance checks for each aircraft

4.2 Illustrative Example problem specifications and parameters

To verify the model formulation explained in the previous section 3, an illustrative example had to be created, to verify the model feasibility. An illustrative example is considerably smaller in size than the case study, but it still has enough data to be a good indicator of the model performance.

For this specific case, the aircrafts considered are only three: an A319, an A320 and an A321. There are 2 different types of checks to be considered, the A-type (light-check) that has 4 different maintenance checks (A1, A2, …, A4), and the C-type (base-check) that has 12 different checks (C1, C2, …, C12). The planning horizon considered is 2 years, which is equivalent to 105 weeks. To begin, the constant values are defined.

Table 4.1 - Constant values for the illustrative example

Constant	Description	Value	Unit
$\boldsymbol{Np}$	Number of planes	3	-
$\boldsymbol{Nc}$	Number of different types of maintenance checks	2	-
$\boldsymbol{Nw}$	Number of weeks in planning horizon	105	Weeks
$\boldsymbol{Nk_1}$	Number of maximum different A-type maintenance checks	4	-
$\boldsymbol{Nk_2}$	Number of maximum different C-type maintenance checks	12	-
$\boldsymbol{M}$	Large number	100,000	-
$\boldsymbol{\varepsilon}$	Small number	0.001	-

The planning horizon considered starts on the week of the 5th of February 2018 and finishes on the week of 3rd of February 2020. Therefore, the high season set can be defined for certain weeks t, which includes summertime and Christmas and new year´s season, as shown in Table 4.2. These ranges of time are assumptions and not direct information from TAP Air Portugal.

Table 4.2 - High Season set values for the illustrative example.

Set	Time			
	18th June 2018 – 16th September 2018	*17th December 2018 – 6th January 2019*	*17th June 2019 – 15th September 2019*	*16th December 2019 – 5th January 2020*
T^{HS}	20 ≤ t ≤ 32	46 ≤ t ≤ 48	72 ≤ t ≤ 84	98 ≤ t ≤ 100

Table 4.3 sets values for the decision weights included in the objective function. The purpose of these values is to provide a different preference on the different components of the objective function, which impact the final optimization value.

Table 4.3 – Values of the decision weights in the objective function for the illustrative example

Parameter	Description	Value
$\gamma 1$	Decision weight for third term of the objective function	0.000001
$\gamma 2$	Decision weight for fourth term of the objective function	0.001

The parameters for the illustrative example are set with information given from TAP M&E and are presented in the following tables. Table 4.4 sets the parameters for the cost of each type of maintenance and for the unavailability of the aircraft when the maintenance check occurs.

Table 4.4 - Cost values of the maintenance checks and unavailability for the illustrative example

Parameter	Description	Value	Unit
c_{un}	Unavailability cost	20	k€
c_1	Cost for A-type maintenance check	30	k€
c_2	Cost for C-type maintenance check	600	k€

In the next table 4.5, the threshold values to trigger each type of maintenance are presented. These values are given from TAP M&E, but they can also be confirmed from the COSCAP Seminar Schedule Maintenance (Airbus, 2013). The flight hours are the elapsed time between wheel lift off and touchdown, while each flight cycle considers a complete take-off and landing sequence. The weeks represent the threshold value of time between checks, which is 4 months for the A-type check and 24 months for the C-type check.

Table 4.5 - Threshold values for flight hours, flight cycles and weeks for the illustrative example

Parameter	**Description**	**Value**	**Unit**
FH_1^{max}	threshold value for maximum flight hours, between two consecutive A-type maintenance checks	750	hour
FC_1^{max}	threshold value for maximum flight cycles, between two consecutive A-type maintenance checks	750	cycle
W_1^{max}	threshold value for maximum weeks, between two consecutive A-type maintenance checks	17	week
FH_2^{max}	threshold value for maximum flight hours, between two consecutive C-type maintenance checks	7,500	hour
FC_2^{max}	threshold value for maximum flight cycles, between two consecutive C-type maintenance checks	5,000	cycle
W_2^{max}	threshold value for maximum weeks, between two consecutive C-type maintenance checks	105	week

The following table 4.6 sets the parameters for the accumulated flight hours, flight cycles and weeks since the last maintenance check, for each type and each aircraft, at the beginning of the time horizon (*t=0*).

Table 4.6 - Accumulated flight hours, flight cycles and weeks for each aircraft of the illustrative example

Plane	Parameters					
	$FH_{1,p}^{0}$	$FC_{1,p}^{0}$	$W_{1,p}^{0}$	$FH_{2,p}^{0}$	$FC_{2,p}^{0}$	$W_{2,p}^{0}$
A319 [1]	258	108	4	6,095	2,914	89
A320 [2]	446	165	6	5,453	2,093	68
A321 [3]	166	64	3	2,818	1,172	55

To better understand the following two tables, Figure 4.3 shows a scheme on how the planning horizon is divided into 3 different time groups. In the Time group 1 are the times (1 ≤ t ≤ 8; 45 ≤ t ≤ 60; 97 ≤ t ≤ 105), in Time group 2 are the times (9 ≤ t ≤ 16; 37 ≤ t ≤ 44; 61 ≤ t ≤ 68; 89 ≤ t ≤ 96) and in Time group 3 are the times (17 ≤ t ≤ 36; 69 ≤ t ≤ 88). These divisions represent, approximately, the Summertime (Time 3), the Fall / Spring times (Time 2) and the Wintertime (Time 1). This division was also done, because the data given from TAP M&E was only for the first 26 weeks, and a symmetric assumption for the rest of the planning horizon had to be made.

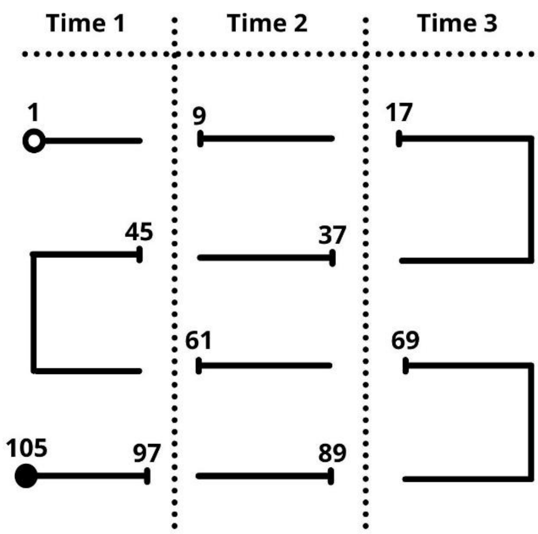


Figure 4.3 - Partition of the time periods (till the planning horizon) into 3 time groups for the illustrative example

Table 4.7 sets the parameters for the estimated flight hours for each aircraft. These values are estimations given from TAP M&E, considering the flights that are scheduled for the first 6 months (26 weeks), and then a symmetric assumption is done for the next 18 months (79 weeks). The table 4.7

represents the flight hours of each type of aircraft, on each group of time, previously defined on Figure 4.3.

Table 4.7 - Weekly flight hours for each aircraft type for the illustrative example

Plane Type	**Parameter**		
	$FHw_{p,t}$		
	Time group 1	Time group 2	Time group 3
A319	66.8	70.0	73.5
A320	77.3	80.5	84.0
A321	74.1	80.5	80.5

The next table 4.8 sets the parameters for the estimated flight cycles for each aircraft. These values are estimations given from TAP M&E, considering the flights that are scheduled for the first 6 months (26 weeks), and then a symmetric assumption is done for the next 18 months (79 weeks). The table 4.8 represents the flight cycles of each type of aircraft, on each group of time, previously defined on Figure 4.3. These values, to be given in the same ranges of time of the flight hours, are given as an average number of the flight cycles done in those ranges of time.

Table 4.8 - Weekly flight cycles for each aircraft type for the illustrative example

Plane Type	**Parameter**		
	$FCw_{p,t}$		
	Time group 1	Time group 2	Time group 3
A319	37.1	38.9	40.7
A320	31.0	32.3	33.6
A321	31.0	32.3	33.6

The table 4.9 represents the hangar availability for each week of the planning horizon. The hangar has a capacity for 5 narrow-body (A319, A320, A321) type of aircraft at the same time. Notice that for each week, only 4 of the 7 days are available for maintenance tasks to be performed. The estimated hangar availability for each week is presented, based on information from TAP M&E.

Table 4.9 - Hangar availability value along the planning horizon for the illustrative example

Time (week)	**Parameter**
	S_t
$1 \leq t \leq 105$	6

The following table 4.10 shows the last type and number of maintenance checks for each aircraft at the beginning of the planning horizon.

Table 4.10 - Last number of maintenance check type for each aircraft at the beginning of the planning horizon for the illustrative example

Plane	**Parameters**	
	$L_{1,p}$	$L_{2,p}$
A319 [1]	1	9
A320 [2]	2	8
A321 [3]	2	8

With all the constants and parameters set and defined, the optimization model is applied to the illustrative example, and the results are presented for verification.

4.3 Results for Exact method, Branch-and-Bound Approach

To verify the model, the results are analysed to confirm that the maintenance checks are done without the threshold values being exceeded and that the aircrafts go to the hangar at the same time, every time a maintenance activity is performed. If everything is confirmed and is accordingly, the model can be considered verified.

For the exact method, branch-and-bound approach, the model converged to an optimal solution with a minimum best value of 3,427.4, as shown in the stats provided by the commercial solver FICO Xpress in table 4.11.

Table 4.11 - Stats of the exact method for the illustrative example

Best Bound	Best Solution	Gap (%)	Status	Time Elapsed (s)
3,427.4	3,427.4	0	Solution is optimal.	82

Since the model is executed with a branch-and-bound approach, and the objective function is to be minimized, there is going to be a lower bound that the algorithm will try to reach, to achieve the best solution for the problem. The optimality gap is the calculation between the best solution minus the best bound divided by the best solution, in percentage, and in this case, the value is 0%, which reflects on the solution being considered optimal.

These values all come from a MIP search done by the programme, that evaluates several possible solutions, while discarding solutions based on the value difference to the lower bound, until it reaches the optimal solution. This search can be seen on Figure 4.4.

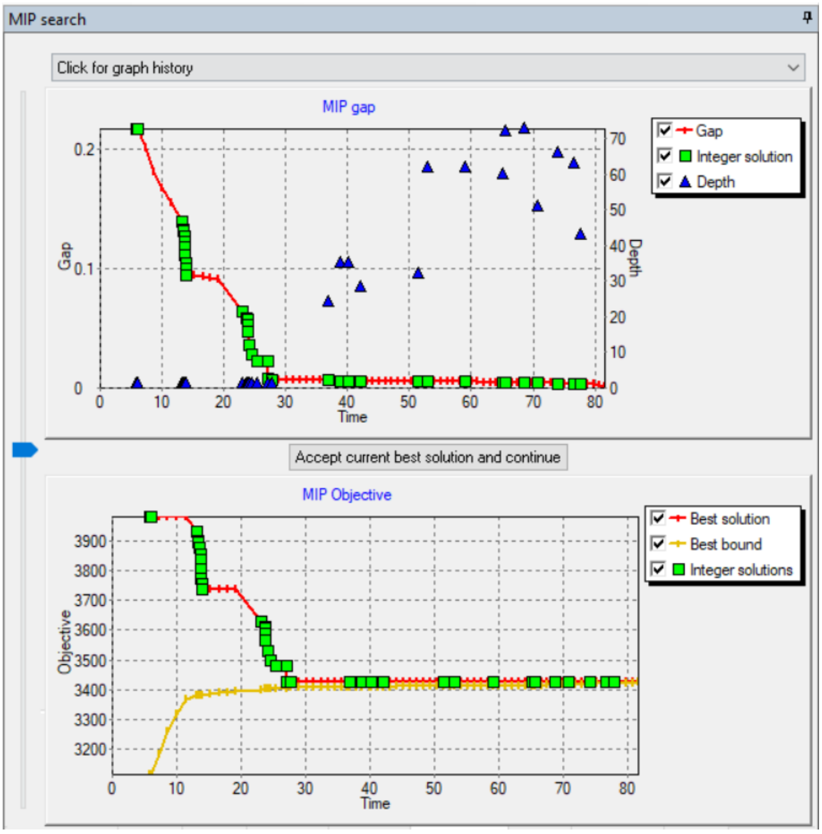


Figure 4.4 - MIP search of the exact method for the illustrative example

Even though the minimum solution value found is equal to 3,427.4, the corresponding out-of-pocket minimum cost is only 3,360 k€. This difference occurs because, for the objective function, only the first and second terms are the real (out-of-pocket) cost of the scheduled maintenance activities, since the third and fourth terms of the objective function serve only as a penalty, as explained in section 3.2.7. The total time elapsed since the beginning of the program computation until the end is 82 seconds, which represents a relatively low time of computation.

Regarding the results, the solution provides important information, such as when maintenance checks should occur, when an aircraft needs to stay at the hangar and the accumulated values of flight hours, flight cycles and weeks throughout the planning horizon.

Firstly, the decision variables $\boldsymbol{x_{p,c,k,t}}$ and $\boldsymbol{y_{p,t}}$, which refer to when a maintenance activity needs to be performed and the aircraft stays at the hangar, respectively, are analysed and checked for any inaccuracies. It is confirmed that whenever a maintenance check occurs, the aircraft goes to the hangar to perform the activity. It can also be seen that for the maintenance C-type, the aircraft stays at the hangar two weeks in a row. Moreover, the model takes the unavailability of the aircraft as an opportunity to also perform an A-type maintenance check at the same time as a C-type.

To better visualize and understand the solution, the optimization model is set to output both a graph that shows when aircrafts go to the hangar throughout the planning horizon (Figure 4.5), and a data file that states for each plane, when a certain maintenance check must be performed.

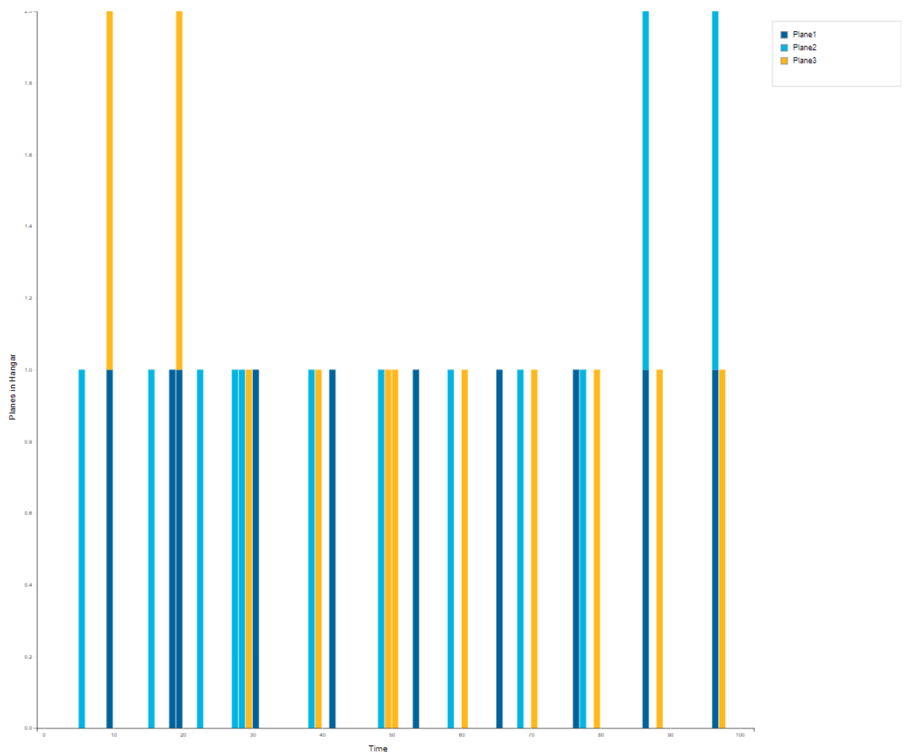


Figure 4.5 - Graph of planes in hangar throughout the planning horizon of the exact method for the illustrative example

To ensure that the model is feasible, and consequently verified, the post processing parameters values are analysed. For each of the three aircrafts in the illustrative example, none exceeds the threshold values of flight hours, flight cycles or time between two sequential same-type checks, at any given time throughout the planning horizon. Since that for this illustrative example results, the threshold values that trigger when a maintenance check needs to occur are the ones related to the accumulated weeks for the C-type maintenance checks and the total flight hours for both types of maintenance. Table 4.12 shows the average values of the total flight hours in the week before an A-type and a C-type maintenance check occurs and the average accumulation of weeks on the week before a C-type maintenance occurs, for each aircraft.

Table 4.12 - Average values of the total flight hours on the week before an A-type and C-type maintenance check occurs and the average accumulation of weeks on the week before a C-type maintenance occurs for the exact method of the illustrative example

Plane	**Post-Processing Parameters**		
	$TFH_{1,p,t}$	$TFH_{2,p,t}$	$TW_{2,p,t}$
	Average (hours)	**Average (hours)**	**Average (weeks)**
A319 [1]	703.8	7,196.5	105
A320 [2]	661.1	7,485.5	93
A321 [3]	695.5	6,534.5	102

Once this is confirmed, the model can be considered feasible and is verified, which results in the associated schedule of maintenance operations, presented in Figure 4.6.

		Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Nº	Type	Registration	5 feb.	12 feb.	19 feb.	26 feb.	5 mar.	12 mar.	19 mar.	26 mar.	2 apr.	9 apr.	16 apr.	23 apr.	30 apr.	7 may	14 may	21 may	28 may	4 jun.	11 jun.	18 jun.	25 jun.	2 jul.	9 jul.	16 jul.	23 jul.	30 jul.
1	A319	CS-TTV									A2									A3+C10								
2	A320	CS-TNP					A3										A4							A1				
3	A321	CS-TJH									A3										A4							

		Week	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
Nº	Type	Registration	6 aug.	13 aug.	20 aug.	27 aug.	3 sep.	10 sep.	17 sep.	24 sep.	1 oct.	8 oct.	15 oct.	22 oct.	29 oct.	5 nov.	12 nov.	19 nov.	26 nov.	3 dec.	10 dec.	17 dec.	24 dec.	31 dec.	7 jan.	14 jan.	21 jan.	28 jan.
1	A319	CS-TTV				A4											A1											
2	A320	CS-TNP	A2+C9											A3										A4				
3	A321	CS-TJH			A1										A2										A3+C9			

		Week	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78
Nº	Type	Registration	4 feb.	11 feb.	18 feb.	25 feb.	4 mar.	11 mar.	18 mar.	25 mar.	1 apr.	8 apr.	15 apr.	22 apr.	29 apr.	6 may	13 may	20 may	27 may	3 jun.	10 jun.	17 jun.	24 jun.	1 jul.	8 jul.	15 jul.	22 jul.	29 jul.
1	A319	CS-TTV	A2												A3											A4		
2	A320	CS-TNP						A1										A2									A3	
3	A321	CS-TJH								A4										A1								

		Week	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Nº	Type	Registration	5 aug.	12 aug.	19 aug.	26 aug.	2 sep.	9 sep.	16 sep.	23 sep.	30 sep.	7 oct.	14 oct.	21 oct.	28 oct.	4 nov.	11 nov.	18 nov.	25 nov.	2 dec.	9 dec.	16 dec.	23 dec.	30 dec.	6 jan.	13 jan.	20 jan.	27 jan.	3 fev.
1	A319	CS-TTV								A1										A2									
2	A320	CS-TNP								A4										A1									
3	A321	CS-TJH	A2									A3									A4								

Figure 4.6 - Maintenance operation schedule of the exact method for the illustrative example

4.4 Results for Heuristic Approach

For the heuristic approach, the formulation explained in section 3.2.9 is added to the model formulation. Similar to the exact method approach, to verify the model, the same process is followed to ensure that the model is feasible.

On the heuristic approach, the model converged to an optimal solution with a minimum best solution value of 3,427.4, as shown in the stats provided by the commercial solver FICO Xpress in Table 4.13. Note that these stats are only for the last aircraft to be executed in the programme, which represents the final solution. The time elapsed is the final total computational time. All the previous aircrafts reached an optimal solution in their MIP search, in order for the programme to move to the next one and so on.

Table 4.13 - Stats of the heuristic approach for the illustrative example

Best Bound	**Best Solution**	**Time Elapsed (s)**
3,427.4	3,427.4	28

Regarding the MIP search, for the model to end its computation, all the aircrafts converged to an optimal solution, which means that all the solutions for each plane reached an equal or nearly equal value to the best bound, consequently having an optimality gap value near zero, for each one. For this illustrative example, the criteria chosen for the order in which the aircrafts were executed, was the amount of weekly flight hours $\boldsymbol{FHw_{p,t}}$, i.e., for this example, the first aircraft to be executed is the A320 [2], then the A321 [3], and finally the A319 [1], knowing the $\boldsymbol{FHw_{p,t}}$ parameter values for each plane, stated in section 4.2.

Similar to the exact method approach, only the first and second term of the objective function are considered for the actual costs of the scheduled maintenance activities, what results in a real minimum cost of 3,360 k€. The total time elapsed since the beginning of the program computation until the end is 28 seconds, which represents a competitive time.

The results are now analysed, to check for any inaccuracies that could make the solution unfeasible. Firstly, the decision variables $x_{p,c,k,t}$ and $y_{p,t}$, which refer to when a maintenance activity needs to be performed and the aircraft stays at the hangar, respectively, are analysed. It is confirmed that whenever a maintenance check occurs, the aircraft goes to the hangar to perform the activity. It can also be seen that for the maintenance C-type, the aircraft stays at the hangar two weeks in a row. Moreover, the model uses the unavailability of the aircraft to perform an A-type maintenance check at the same time as a C-type. Although there is a criterion for the order in which the model executes each plane, already explained above, this does not mean that for the results the order is affected, i.e., for the results, plane 1 is still A319 [1], plane 2 is still A320 [2], and plane 3 is still A321 [3].

To better visualize and understand the solution, the programme is set to output both a graph that shows when aircrafts go to the hangar throughout the planning horizon (Figure 4.7), and a data file that states for each plane, when a certain maintenance check must be performed.

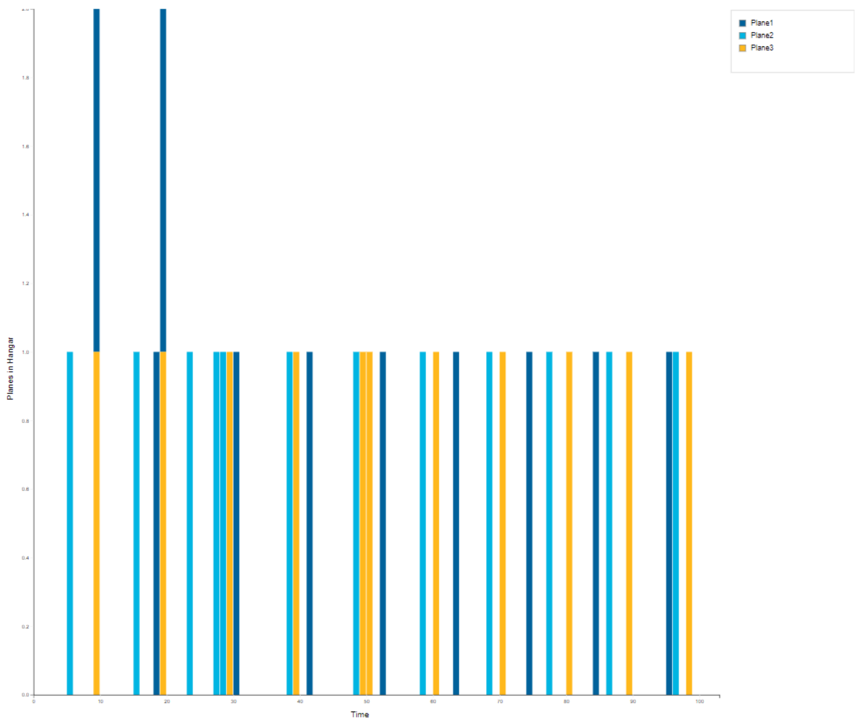


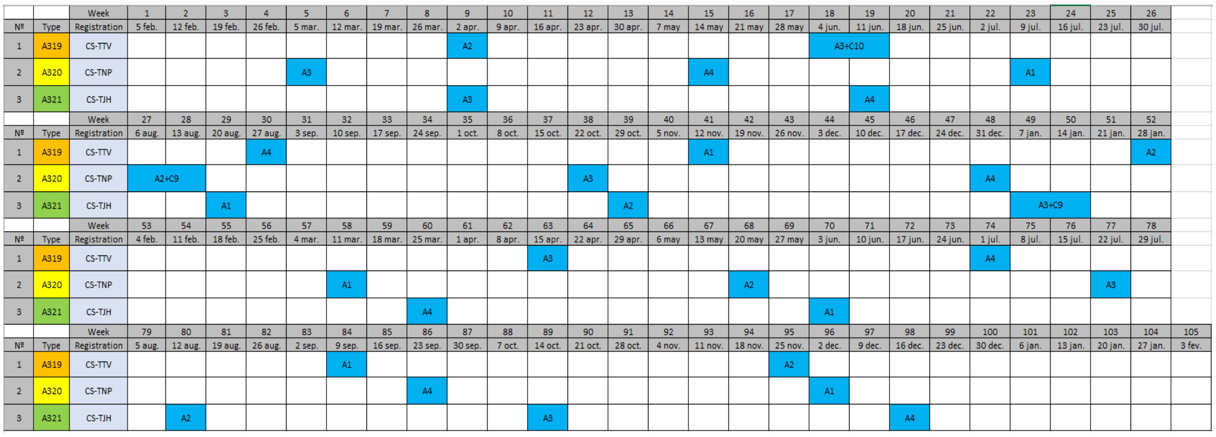
Figure 4.7 - Graph of planes in hangar throughout the planning horizon of the heuristic approach for the illustrative example

To ensure that the model is feasible, and consequently verified, the post processing parameters values are analysed. For each of the three aircrafts in the illustrative example, none exceeds the threshold values for the flight hours, flight cycles or time between two sequential same-type checks, at any given time throughout the planning horizon. Since that for this illustrative example results, the threshold values that dictate when a maintenance check needs to occur are the ones related to the accumulated weeks for the C-type maintenance checks and the total flight hours for both types of maintenance. Table 4.14 shows the average values of the total flight hours in the week before an A-type and C-type maintenance check occurs and the average accumulation of weeks on the week before a C-type maintenance occurs, for each aircraft.

Table 4.14 - Average values of the total flight hours on the week before an A-type and C-type maintenance check occurs and the average accumulation of weeks on the week before a C-type maintenance occurs for the heuristic approach of the illustrative example

Plane	**Post-Processing Parameters**		
	$TFH_{1,p,t}$	$TFH_{2,p,t}$	$TW_{2,p,t}$
	Average (hours)	**Average (hours)**	**Average (weeks)**
A319 [1]	696.1	7,196.5	105
A320 [2]	661.1	7,485.5	93
A321 [3]	702.9	6,534.5	102

Once this is confirmed, the model can be considered feasible and is verified, which results in the maintenance operations schedule, presented in Figure 4.8.



		Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Nº	Type	Registration	5 feb.	12 feb.	19 feb.	26 feb.	5 mar.	12 mar.	19 mar.	26 mar.	2 apr.	9 apr.	16 apr.	23 apr.	30 apr.	7 may	14 may	21 may	28 may	4 jun.	11 jun.	18 jun.	25 jun.	2 jul.	9 jul.	16 jul.	23 jul.	30 jul.
1	A319	CS-TTV									A2									A3+C10								
2	A320	CS-TNP					A3										A4								A1			
3	A321	CS-TJH									A3										A4							

		Week	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
Nº	Type	Registration	6 aug.	13 aug.	20 aug.	27 aug.	3 sep.	10 sep.	17 sep.	24 sep.	1 oct.	8 oct.	15 oct.	22 oct.	29 oct.	5 nov.	12 nov.	19 nov.	26 nov.	3 dec.	10 dec.	17 dec.	24 dec.	31 dec.	7 jan.	14 jan.	21 jan.	28 jan.
1	A319	CS-TTV				A4											A1											A2
2	A320	CS-TNP	A2+C9											A3										A4				
3	A321	CS-TJH			A1										A2										A3+C9			

		Week	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78
Nº	Type	Registration	4 feb.	11 feb.	18 feb.	25 feb.	4 mar.	11 mar.	18 mar.	25 mar.	1 apr.	8 apr.	15 apr.	22 apr.	29 apr.	6 may	13 may	20 may	27 may	3 jun.	10 jun.	17 jun.	24 jun.	1 jul.	8 jul.	15 jul.	22 jul.	29 jul.
1	A319	CS-TTV											A3											A4				
2	A320	CS-TNP						A1										A2									A3	
3	A321	CS-TJH								A4										A1								

		Week	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Nº	Type	Registration	5 aug.	12 aug.	19 aug.	26 aug.	2 sep.	9 sep.	16 sep.	23 sep.	30 sep.	7 oct.	14 oct.	21 oct.	28 oct.	4 nov.	11 nov.	18 nov.	25 nov.	2 dec.	9 dec.	16 dec.	23 dec.	30 dec.	6 jan.	13 jan.	20 jan.	27 jan.	3 fev.
1	A319	CS-TTV						A1											A2										
2	A320	CS-TNP								A4										A1									
3	A321	CS-TJH		A2									A3									A4							

Figure 4.8 - Maintenance operation schedule of the heuristic approach for the illustrative example

4.5 Comparison of results for the Illustrative Example

After the presentation of the results, a comparison between both approaches can be made. The first aspects that need to be compared is the real total minimum cost and the total time elapsed for the model computation on FICO Xpress.

Table 4.15 - Comparison between the exact method and the heuristic approach for the illustrative example

	Exact method approach	**Heuristic approach**
Best solution value	3,427.4	3,427.4
Real total minimum cost (k€)	3,360	3,360
Total time elapsed (s)	82	28

From Table 4.15, it can be observed that the real objective solution values are equal for both approaches, while for the total time elapsed the difference is considerable, since there is an approximately 66% decrease in the computational time elapsed, from the exact method to the heuristic approach.

Although the exact method approach is a more reliable approach, since it takes into consideration all the aircrafts at the same time and not one-by-one in an order defined by the user, the heuristic could still achieve the same optimal solution value as the exact method, which provides a simple but good indicator of the performance of this optimization approach. As for the time elapsed, the value for the heuristic being lower than the exact method goes accordingly to what it was expected by doing the heuristic method.

Regarding the schedule itself, both approaches reach a similar maintenance operation schedule, having just some minor differences, like having one maintenance check done a day before or after than the schedule of the other approach, as shown in Figures 4.6 and 4.8. This concludes that both approaches are verified and feasible, and both are to be taken into consideration and used, to execute and study the larger case study in section 5.

5 Case Study - TAP Air Portugal, Results and Discussion

This section, firstly, introduces and describes the case study of TAP Air Portugal. Afterwards, all the respective specifications and parameters are set and explained in detail, to provide a better and broader understanding of the problem. Furthermore, the results for the case study are presented and discussed. The results from both approaches (the exact method using the branch-and-bound approach and the heuristic approach) are shown. Then, the results are analysed and compared in order to take conclusions of the pros and cons of both approaches. Finally, several tests on the model formulation and parameter values are conducted, and consequently a sensitivity analysis study is performed.

5.1 TAP Air Portugal Problem Specifications

The focus of this case study is to optimize the maintenance planning schedule in a way that it reduces costs and increases aircraft availability. As stated before, the aviation industry is an extremely competitive market, and consequently, optimization in maintenance operations is very important for airline companies' finances and competitiveness, while still ensuring that aircrafts comply with strict safety regulations. For this problem, several aspects must be considered as detailed in sections 3 and 4, such as: the maintenance hangar facilities, the airline aircraft fleet and flights information, the aircrafts maintenance regulations and the planning horizon, including variations on the type of season.

The airline company that is under analysis is TAP Air Portugal, but since this is a problem related to maintenance operations, almost all data came from TAP Maintenance & Engineering (TAP M&E), which is the aircraft maintenance and engineering unit of TAP Air Portugal.

For this case study, the data used is from 2018, which means that the aircraft fleet and time estimations are from that year (fleet considered for this dissertation is different from the fleet present in table 1.1). The data corresponds to the same used in Martinho (2018) model, because more recent data and information could not be acquired. The data was only for a 6-month period, but for this case study, symmetric assumptions are done, in order to schedule it for a 2-year planning horizon, i.e., the data related to the weekly flight hours and weekly flight cycles, for the first 6 months, is then symmetric for the second 6 months, and the same happens for the third and fourth 6-month periods. These assumptions will be further explained in detail in section 5.2.

From the maintenance schedule provided by TAP M&E in Martinho (2018), the hangar capacity was assumed, by analysing the maximum number of aircrafts that were in the hangar at a certain week, from the TAP M&E schedule, and then assuming that value for every week of the planning horizon. As previously stated on section 1, the hangar facilities in Humberto Delgado Airport (Lisbon, Portugal) includes 3 hangars that can simultaneously hold 3 WB (wide-body aircraft) and 5 NB (narrow-body aircraft), and this was also taken into account for the assumed value. Variations on the hangar capacity throughout the planning horizon are not considered, since further information on this could not be obtained and variations in the availability of human resources is not considered in this present dissertation. A further explanation on this will be given in section 5.2.

Although TAP Air Portugal aircraft fleet includes *narrow-body* and *wide-body*, this case study only studies and schedules the *narrow-body* part of the fleet, which means only the Airbus aircrafts A319, A320 and A321 are considered.

In short, the main objective of TAP Air Portugal is to schedule the maintenance checks of their fleet of 45 aircrafts, in a 2-year planning horizon, by using their hangar facilities on the Lisbon Airport, while minimizing maintenance and unavailability costs, and avoiding C-type maintenance checks during the High Season.

The specifications for the present case study are now discussed below. In the appendix, the aircrafts present in this case study are given and associated with a plane number, which is used to identify them on the results. In the appendix A1, A2 and A3, the fleet descriptions (divided by aircraft type: the A319, the A320 and the A321, respectively) are provided.

The following Table 5.1, sets the values for the constants in the case study, which only differs from the illustrative example on the number of planes from 3 to 45.

Table 5.1 - Constant values for the Case Study

Constant	Description	Value	Unit
Np	Number of planes	45	-
Nc	Number of different types of maintenance checks	2	-
Nw	Number of weeks in planning horizon	105	Weeks
Nk_1	Number of maximum different A-type maintenance checks	4	-
Nk_2	Number of maximum different C-type maintenance checks	12	-
M	Large number	100,000	-
ε	Small number	0.001	-

5.2 Parameters for the TAP Case Study

In this section the parameters are presented. All the parameters are the same as the ones presented in section 4.2, with the exceptions of the increase in the number of aircrafts considered, and their individual parameters of accumulated flight hours, flight cycles and weeks, and their last number of maintenance check.

The appendix A4, A5 and A6, set the parameters for the accumulated flight hours, flight cycles and weeks since the last maintenance check, on the beginning of the time horizon (*t=0*), for each type and each aircraft (divided by aircraft type: the A319, the A320 and the A321, respectively).

The next Tables 5.2, 5.3 and 5.4, shows the last type and number maintenance checks for each aircraft at the beginning of the planning horizon, divided in three tables for each type of aircraft (A319, A320 and A321, respectively).

Table 5.2 - Last number of maintenance check type for each A319 aircraft at the beginning of the planning horizon for the Case Study

Parameter	Plane																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
$L_{1,p}$	1	3	1	1	3	4	4	3	2	2	4	1	1	1	2	1	1	1	1	1	1
$L_{2,p}$	1	1	1	12	12	1	12	12	12	12	12	11	11	11	11	11	12	9	9	9	9

Table 5.3 - Last number of maintenance check type for each A320 aircraft at the beginning of the planning horizon for the Case Study

Parameter	Plane																			
	22	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**
$L_{1,p}$	4	3	3	3	4	2	2	1	4	2	1	3	2	1	1	1	3	1	4	1
$L_{2,p}$	10	12	12	12	11	11	11	9	9	8	5	5	5	5	4	4	7	7	3	12

Table 5.4 - Last number of maintenance check type for each A321 aircraft at the beginning of the planning horizon for the Case Study

Parameter	Plane			
	42	**43**	**44**	**45**
$L_{1,p}$	2	1	1	2
$L_{2,p}$	11	11	10	8

5.3 Results of the Case Study

Since the main objective of this dissertation is to reach a solution for the case study, the results need to be presented and explained in detail. The results from both approaches (the exact method using the branch-and-bound approach and the heuristic approach) are explored in the following sections. The model computations and respective solutions were done on a laptop with an Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz 2.81 GHz processor with a RAM of 16 GB.

5.3.1 Results using the Branch-and-Bound Approach (exact method)

In this subsection the exact method (branch-and-bound approach) are presented and analysed. For the exact method, the model was executed for 199,662 seconds, which is approximately 55 hours and 28 minutes. This approach could not find any solution during this time, as shown in the stats provided by the commercial solver FICO Xpress in Table 5.5.

Table 5.5 - Stats of the exact method approach for the Case Study

Best Bound	Best Solution	Gap (%)	Status	Time Elapsed (s)
48,232	-	-	Searching for first integer solution	199,662

The “exact method” approach could not find any solution during this period, which is considered too long for this type of problem, and thus, no more results are discussed in this subsection 5.3.1. Note that the increase in the size of the problem from 3 aircrafts (in the illustrative example) to 45 aircrafts (in the case study) is the main reason why the exact method does not achieve any solution, even for a computational time of 55 hours and 28 minutes. It means that more time would be needed for the exact method approach to reach a feasible solution. Nonetheless, a comparison between this approach and the heuristic approach will be analysed in section 5.4, regarding the evolution of computational time until a solution is achieved and the solution itself, between both approaches.

5.3.2 Results of the Heuristic Approach

In this subsection, the results for the heuristic approach applied to the case study are presented and analysed. Several aspects will be thoroughly verified.

For the case study, and similar to the illustrative example, the criteria chosen for the order in which the aircrafts were executed, was the amount of weekly flight hours $\boldsymbol{FHw_{p,t}}$, i.e., for the case study, the first aircrafts to be executed are the A320´s from aircraft number 22 to number 41, which are the ones with higher weekly flight hours throughout the planning horizon, then the A321´s from aircraft number 42 to number 45, and finally the A319´s from aircraft number 1 to number 21, knowing the $\boldsymbol{FHw_{p,t}}$ parameter values for each plane, present in Table 4.7.

On the heuristic approach, the model converged to an optimal solution with a minimum solution value of 53,130.3, as shown in the stats provided by the commercial solver FICO Xpress in Table 5.6. Note that these stats are only for the last aircraft to be executed in the programme, which represents the totality of the problem. All the previous aircrafts reached an optimal solution in their MIP search, in order for the programme to move to the next aircraft and so on.

Table 5.6 - Stats of the heuristic approach for the Case Study

Best Solution	**Time Elapsed (s)**
53,130.3	2,886

Only the first and second term of the objective function are considered for the actual costs of the scheduled maintenance activities, which results in a real minimum cost of 52,290 k€.

Regarding the MIP search, for the model to end its computation, all the aircrafts converged to an optimal solution, which means that all the solutions for each plane reached an equal or nearly equal value to the best bound, consequently having an optimality gap value near zero, for each one. Figure 5.1 shows the MIP solving time for each aircraft, i.e., the time from starting to search for an integer solution, until it finds the optimal solution.

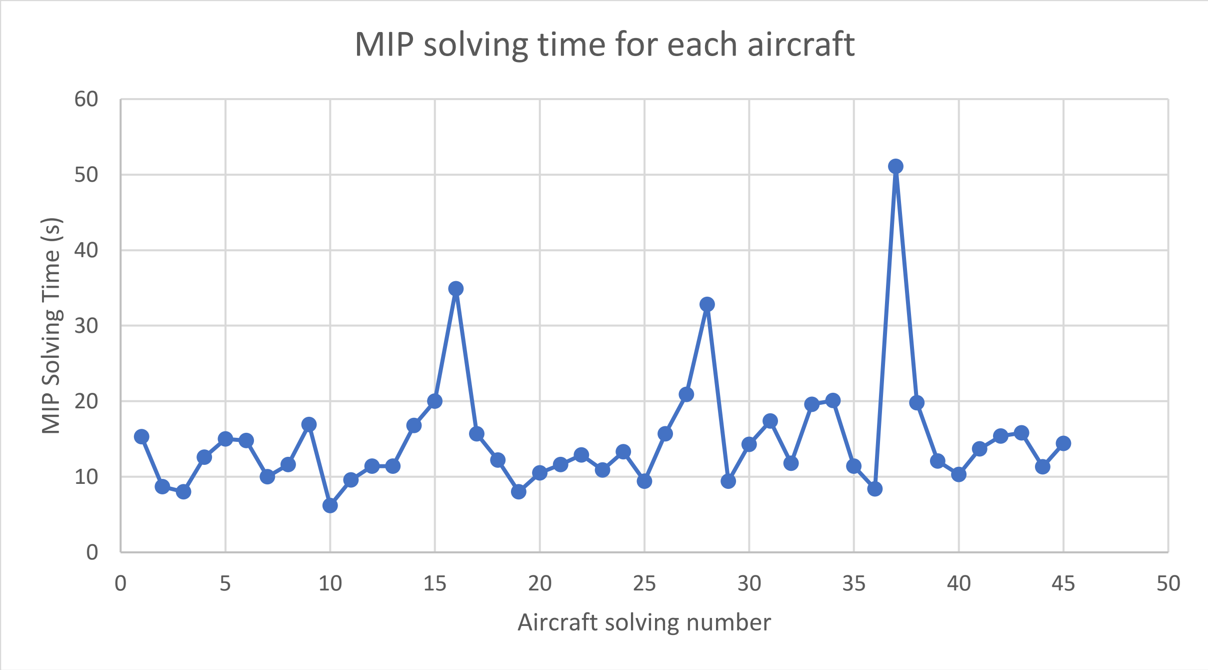


Figure 5.1 - MIP solving time for each aircraft for the heuristic approach of the case study

From this figure, we can calculate the average value of MIP solving time, which results in 15 seconds approximately. This means that only 675 seconds are used to solve the MIP search for the entire aircraft fleet.

Even though the total MIP search time is 675 seconds, the problem needs to import increasing data due to the write or rewrite of arrays from previous solutions, each time it moves to the next aircraft, which justifies a larger overall computational time throughout the number of aircrafts. This can be verified in Figure 5.2, which shows the total computational solving time for each aircraft, since the end of the previous aircraft until it finds an optimal solution, increases along with the number of aircrafts already solved. Figure 5.3 also gives an overview on the evolution of computation time throughout the heuristic procedure. The total time elapsed since the beginning of the program computation until the end is 2,886 seconds, which represents a competitive time of approximately 48 minutes.

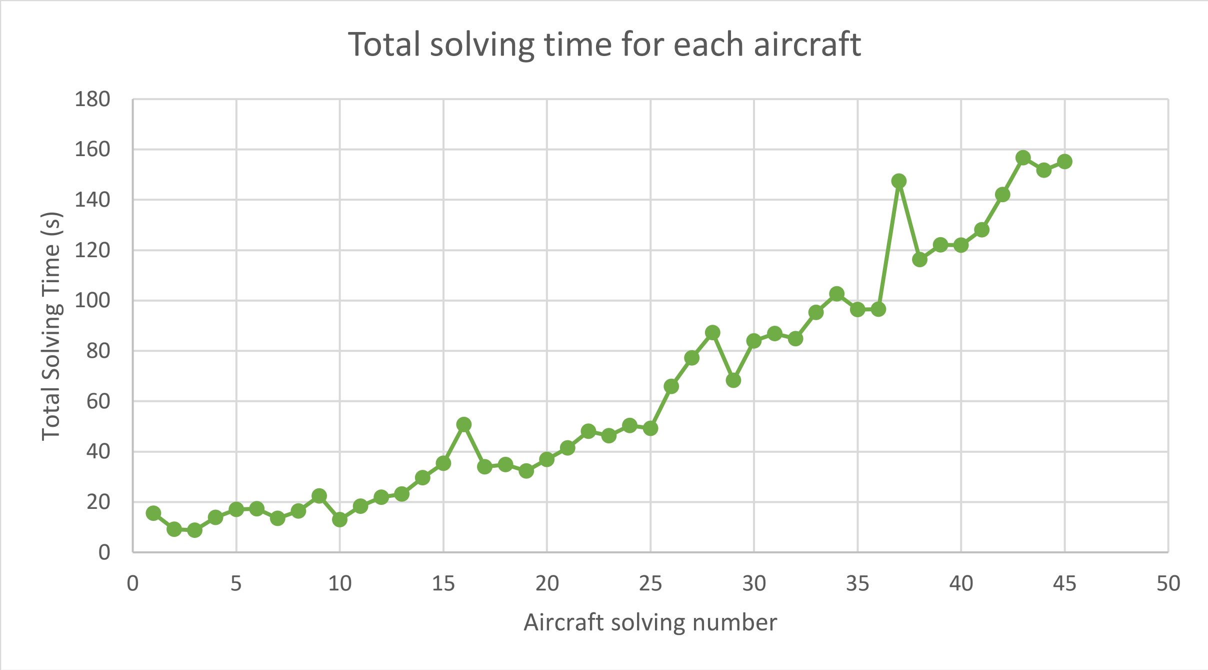


Figure 5.2 - Total solving time for each aircraft for the heuristic approach of the case study

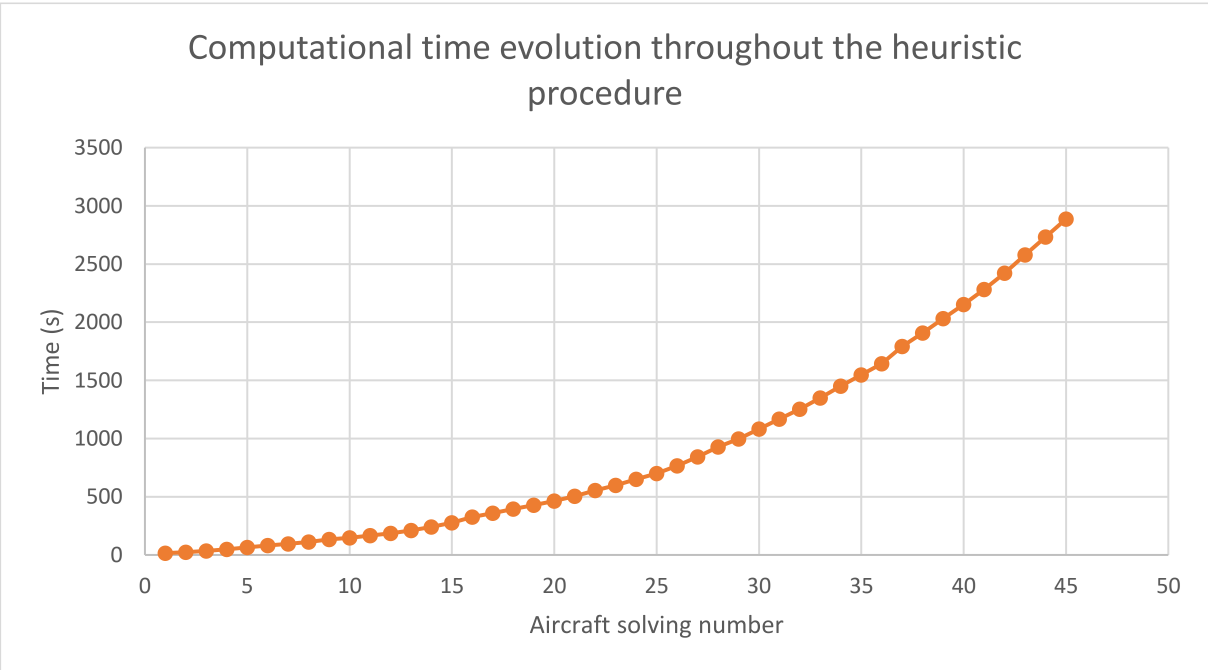


Figure 5.3 - Computational time evolution throughout the heuristic procedure of the case study

The results are now analysed, to check for any inaccuracies that could make the solution unfeasible. Firstly, the decision variables $x_{p,c,k,t}$ and $y_{p,t}$, which refer to when a maintenance activity needs to be performed and the aircraft stays at the hangar, respectively, are analysed and it is confirmed that whenever a maintenance check occurs, the aircraft goes to the hangar to perform the activity. It is also confirmed that for the maintenance C-type, the aircraft stays at the hangar two weeks in a row and that the model also utilizes the unavailability of the aircrafts to also perform an A-type maintenance check at the same time as a C-type.

The programme outputs a graph, but since for 45 aircrafts, the graph becomes too complex to observe, in comparison to the graphs showed in subsections 4.3 and 4.4, a new graph was made to better visualize and understand the solution. The graph on Figure 5.4 shows the variation of the number of aircrafts at the hangar to perform maintenance checks, throughout the planning horizon.

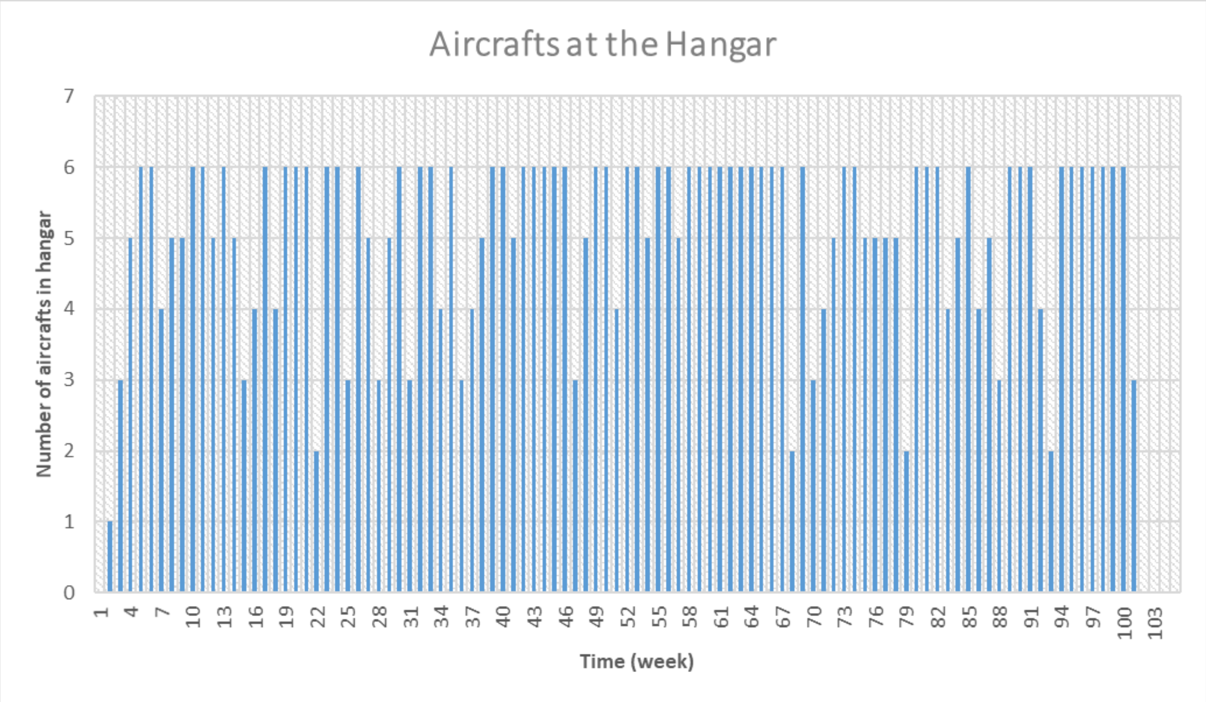


Figure 5.4 - Graph of number of aircrafts in hangar throughout the planning horizon of the heuristic approach for the case study

To ensure that the model is feasible, and consequently verified, the post processing parameters values are analysed. For each of the forty-five aircrafts in the case study, none exceeds the threshold values of flight hours, flight cycles or time between two sequential same type checks, at any given time throughout the planning horizon. Since that for this case study results, the threshold values that dictate when a maintenance check needs to occur are the ones related to the accumulated weeks for the C-type maintenance checks and the total flight hours for both types of maintenance, the appendix A7 demonstrates the average values of total flight hours on the week before an A-type and C-type maintenance check occurs and the average accumulation of weeks on the week before a C-type maintenance occurs, for each aircraft.

Once this is confirmed, the model can be considered feasible and is verified, which results in the maintenance operations schedule for all the 45 aircrafts fleet, throughout the planning horizon of 2-years, given in appendix A8.

For all the times an aircraft is unavailable due to C-type maintenance checks (summing a total of 94 times), only 10 out of the 94 times (which is the sum of $\boldsymbol{y_{p,t}} = \mathbf{1}$, when a C-type maintenance occurs, throughout the planning horizon), are done in weeks of the High Season period. This means that approximately 89.36% of C-type maintenance actions are done out of the High Season, which is aligned with the availability strategy to avoid heavy maintenance checks during high demand peaks.

In Table 5.7, it is given an overview on the cost components influence on the real total minimum cost of 52,290 k€.

Table 5.7 - Influence of cost components on the real total minimum cost of the heuristic approach for the case study

Cost component	Value (k€)	Number of occurrences	Percentage of the real total cost (%)
$\boldsymbol{c_{un}}$	20	510	19.51
$\boldsymbol{c_1}$	30	463	26.56
$\boldsymbol{c_2}$	600	47	53.93

This table shows that the C-type maintenance check cost is what most influences the real total minimum cost, while the unavailability cost only represents approximately one fifth of the total cost. This table also provides an overall view on the total number of times the aircrafts are unavailable (510), the total number of A-type maintenance checks (463) and the total number of C-type maintenance checks (47) that need to occur during the planning horizon of 2 years.

5.4 Analysis and comparison of results for the case study

In this section, the results from both approaches are compared. The model computations and respective solutions were done on the same laptop as the one stated on section 5.3.

Since the exact method could not achieve any feasible solution for the long computational time (stated previously in section 5.3.1), the comparison of both approaches will be based on smaller size problems, with increasing number of aircrafts, i.e., the comparison will be done for 5, 10, 15, 20, 25 and 30 aircrafts. This will show the evolution of the computational time required to achieve a solution, but it will also allow to compare the solutions themselves from both approaches. Since the heuristic approach solves the problem approximately, the heuristic might not achieve the optimal solution, as the exact method approach can if it runs for the computational time needed. Therefore, this comparison will consider both the achieved solution and the computational time needed to compute it.

For this comparison, the aircrafts are chosen randomly from the 45 aircrafts, though they are the same for both approaches, i.e., for the 5 aircrafts analysed, the data of these 5 aircrafts is the same for both the heuristic and exact method approaches. The parameters used for this comparison are the same as the ones used in the case study, differing only on which aircrafts are chosen and the constant $\boldsymbol{Np}$ for the number of aircrafts.

- $\boldsymbol{Np = 5}$ aircrafts numbers are: 10, 21, 25, 31, 45
- $\boldsymbol{Np = 10}$ aircrafts numbers are: 1, 2, 3, 4, 5, 22, 23, 24, 25, 42
- $\boldsymbol{Np = 15}$ aircrafts numbers are: 1, 2, 3, 4, 5, 6, 7, 22, 23, 24, 25, 26, 27, 28, 42
- $\boldsymbol{Np = 20}$ aircrafts numbers are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 22, 23, 24, 25, 26, 27, 28, 29, 30, 42, 43
- $\boldsymbol{Np = 25}$ aircrafts numbers are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43
- $\boldsymbol{Np = 30}$ aircrafts numbers are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 42, 43, 44

Table 5.8 shows the values for the solution and elapsed computational time of each method, for an increasing number of aircrafts.

Note that the solution value of the exact method, is the best solution reached. If the exact method does not reach an optimal solution, the MIP search is analysed, and the best solution considered will be the last solution before the model computation is stopped, which is when it reaches a defined maximum computational time of 12 hours. In this case the optimality gap value is given for consideration. Also, the criteria for the order of the aircrafts in the heuristic approach is the same as in both the case study and illustrative example.

Table 5.8 - Comparison between exact method and heuristic approach for an increasing number of aircrafts

Number of aircrafts Np	Exact method approach				Heuristic approach		
	Best Solution Value	Optimality gap (%)	Solution cost (k€)	Elapsed Time (s)	Best Solution Value	Solution cost (k€)	Elapsed Time (s)
$\boldsymbol{Np = 5}$	5,699.1	-	5,600	39,371	5,699.1	5,600	53
$\boldsymbol{Np = 10}$	11,279.2	1.10	11,100	43,200	11,280.4	11,100	134
$\boldsymbol{Np = 15}$	17,874.5	6.60	17,560	43,200	17,120.6	16,820	276
$\boldsymbol{Np = 20}$	27,784.1	21.02	27,340	43,200	23,328.8	22,970	632
$\boldsymbol{Np = 25}$	53,765.7	49.65	53,260	43,200	28,926.2	28,470	680
$\boldsymbol{Np = 30}$	-	100	-	43,200	34,691.7	34,120	966

Note that all the solutions were confirmed for feasibility. This comparison shows that the heuristic approach achieves the optimal solution for a low number of planes ($\boldsymbol{Np} = \mathbf{3}$ (illustrative example) and $\boldsymbol{Np} = \mathbf{5}$). Moreover, in the latter cases, the heuristic finds better solutions than the best feasible solution found by the exact method, in a much lower computational time needed. Therefore, the heuristic approach can be considered a reasonable method to solve the problem in practice. Again, note that the reduction in the total elapsed computational time is quite visible, which represents benefits in practice when comparing the heuristic approach with the exact method.

5.5 Sensitivity Analysis

In this section, some sensitivity analyses are performed to explore better the impact of changing different parameter values or removing constraints from the mathematical model presented in section 3. From the comparison made in the previous section, the following sensitivity analyses will be performed only for the heuristic approach and the criteria for the order of computation of the aircrafts, is the same as the one used for the case study. The model computations and respective solutions were done on the same laptop as the one stated on section 5.3.

5.5.1 Threshold and weekly parameters values relations

For this first analyse, the relation between the flight hours, flight cycles and accumulation of weeks, and its influence on the model is studied. This study is conducted with the objective of checking the possibility of reducing the problem formulation and data, in order to get better results in terms of time and solution itself, while taking into account the problem specifications and parameters.

From the values given on Table 4.5, a ratio between the thresholds can be calculated (e.g. $\frac{FH_1^{max}}{FC_1^{max}} = \frac{750}{750}$ or $\frac{FC_2^{max}}{W_2^{max}} = \frac{5000}{105}$). These calculations are demonstrated in Table 5.9, where besides that, comments and analyses on the calculations are given. The weekly parameters ratios, also shown in Table 5.9, stand as a decision factor for when analysing the input data from the aircrafts fleet, decide if the problem only needs to consider certain parameters of flight hours, flight cycles and/or weeks, between each type of maintenance check, thus avoiding the use of too many parameters.

Table 5.9 - Ratios and comments between threshold values and weekly flight hours, flight cycles and accumulation of weeks

Threshold's ratio	**Weekly parameters ratio**	**Comments**
$\frac{FH_1^{max}}{FC_1^{max}} = 1\ (\frac{flight\ hours}{flight\ cycle})$ ^ $\frac{FH_2^{max}}{FC_2^{max}} = 1.5\ (\frac{flight\ hours}{flight\ cycle})$	$\frac{FHw_{p,t}}{FCw_{p,t}} > 1.5\ (\frac{flight\ hours}{flight\ cycle})$	If for every aircraft this is true, then the problem does not need to consider $FCw_{p,t}$ for the model formulation
	$\frac{FHw_{p,t}}{FCw_{p,t}} < 1\ (\frac{flight\ hours}{flight\ cycle})$	If for every aircraft this is true, then the problem does not need to consider $FHw_{p,t}$ for the model formulation
$\frac{FH_1^{max}}{W_1^{max}} = 44.12\ (\frac{flight\ hours}{week})$ ^ $\frac{FC_1^{max}}{W_1^{max}} = 44.12\ (\frac{flight\ cycles}{week})$	$FHw_{p,t} > 44.12\ (\frac{flight\ hours}{week})$	If for every aircraft this is true for both weekly parameters ratio, then the problem does not need to consider accumulation of weeks for the A-type checks for the model formulation. Of course, if $FCw_{p,t}$ or $FHw_{p,t}$ is not considered, the same applies for these values
	$FCw_{p,t} > 44.12\ (\frac{flight\ cycles}{week})$	
$\frac{FH_2^{max}}{W_2^{max}} = 71.43\ (\frac{flight\ hours}{week})$ ^ $\frac{FC_2^{max}}{W_2^{max}} = 47.62\ (\frac{flight\ cycles}{week})$	$FHw_{p,t} > 71.43\ (\frac{flight\ hours}{week})$	If for every aircraft this is true for both weekly parameters ratio, then the problem does not need to consider accumulation of weeks for the C-type checks for the model formulation. Of course, if $FCw_{p,t}$ or $FHw_{p,t}$ is not considered, the same applies for these values
	$FCw_{p,t} > 47.62\ (\frac{flight\ cycles}{week})$	

From these comments and analysing the Tables 4.7 and 4.8, it can be concluded that this case study only needs to consider weekly flight hours ($FHw_{p,t}$) and the accumulation of weeks for the C-type maintenance checks. This happens because the ratio of $\frac{FHw_{p,t}}{FCw_{p,t}}$ is higher than 1.5 flight hours per flight cycle, for every type of aircraft, throughout the entire horizon (e.g. $\frac{FHw_{p,t}}{FCw_{p,t}} = \frac{66.8}{37.1} = \mathbf{1.8}$ for an A319 in time group 1), and the $\boldsymbol{FHw_{p,t}}$ is lower than 71.43 flight hours per week for the A319 type of aircrafts, which has an average weekly flight hours value of approximately 70.36 flight hours per week, throughout the planning horizon. Note that, as explained on the comments of Table 5.9, even though from Table 4.8 it can be seen that $\boldsymbol{FCw_{p,t}}$ is lower than 44.12 and 47.62 flight cycles per week for all types of aircraft throughout the planning horizon, since that $\frac{FHw_{p,t}}{FCw_{p,t}}$ is higher than 1.5 flight hours per flight cycle, the flight cycles are immediately discarded as a decision factor for a maintenance check to occur.

Knowing this, a sensitivity analysis is done for a model considering only these parameters, which means not inputting the data related to the weekly flight cycles ($\boldsymbol{FCw_{p,t}}$) and the accumulated parameters since last checks related to flight cycles ($\boldsymbol{FC^0_{c,p}}$) and A-type weeks ($\boldsymbol{W^0_{1,p}}$). Besides this, the constraints **(4)**, **(5)** for $c = 1$, **(9)-(11)**, **(12)-(14)** for $c = 1$, and the post-processing expressions **(30)-(31)**, **(32)-(33)** for $c = 1$, referenced in section 3, are removed from the model formulation. Since that from the original case study heuristic approach, the maximum MIP search time is 51.1 seconds (as seen on Figure 5.1), and for this analysis the heuristic gets stuck on a specific aircraft that cannot achieve the optimal solution in the associated cycle, it was necessary to include a limitation of time for each heuristic cycle, corresponding to a MAX MIP search time of 52 seconds. Adding this limitation of computational time for each heuristic cycle, led to two aircrafts that could not reach an optimal solution.

Once this is done, the model is executed using the heuristic approach, and the results are as follows:

Table 5.10 - Results for the flight hours and C-type weeks only approach for a sensitivity analysis

Real minimum cost solution (k€)	Elapsed time (s)
52,490	2,474

From Table 5.10, it can be seen a reduction of approximately 412 seconds on the elapsed time in comparison to the case study, but for the real minimum cost solution the value increases by 200 k€. This can be justified by the 2 aircrafts that could not reach an optimal solution and could only achieve an optimality gap of 2.82% and 5.30%, in the maximum MIP time of 52 seconds, by analysing the MIP search of every single aircraft. From this study it can be concluded that, even though this specific case study only requires to consider limits for flight hours and weeks for C-type maintenance checks, this model formulation and the solver thrive on having more constraints to help the computation of the optimal solutions.

5.5.2 COVID-19 pandemic situation

Another sensitivity analysis that is going to be studied is the impact of the COVID-19 pandemic on the maintenance scheduling of the aircrafts. The objective is to analyse the performance of the model when the parameters of weekly flight hours and flight cycles are drastically reduced.

To achieve this, for all the fleet, it is assumed that each aircraft only does 2 flight cycles per week, which represent an only go and back flight each week. For the flight hours, it is assumed that for the entire fleet, each aircraft has 2 flight hours per flight cycle. These values are assumed for the entire planning horizon of 2 years, without considering different time groups, and are presented in Table 5.11.

Table 5.11 - Weekly flight hours and flight cycles parameters for the covid situation analysis

	$FHw_{p,t}$	$FCw_{p,t}$
Entire fleet for the entire planning horizon	4	2

Similar to the previous sensitivity analysis, the problem gets stuck on an aircraft that cannot achieve an optimal solution, and so a MAX MIP search time of 52 seconds was set.

Once this is done, the sensitivity analysis is executed, and the results are as follows in Table 5.12.

Table 5.12 - Results for the covid situation sensitivity analysis

Real minimum cost solution (k€)	**Time Elapsed (s)**	**Number of occurrences**		
		Unavailability	**A-type check**	**C-type check**
41,790	3,663	324	277	45

Even though the model reached a total cost value of 41,790 k€, which represents a decrease of 10,500 k€ in the total cost, the model shows an increase of approximately 27% in computational time, in comparison with the case study. Regarding the cost reduction, it goes along with the expectation of drastically reducing the weekly flight hours and cycles, which means that the only decision factor to perform a maintenance activity is the time between maintenance checks. This reflects on the reduction of unavailability by 36.5%, of A-type checks by 40% and of C-type checks by 4%, approximately, which explains the reduction of approximately 20% in the solution cost, in comparison with the case study.

Besides the conclusions for this sensitivity analysis, note that it was assumed that every aircraft has 2 flight cycles per week, which for the COVID-19 pandemic situation may not represent reality, since aircrafts may be grounded for a long uninterrupted time, and the present model does not consider this.

This means that the value of the solution cost may be lower, since the maintenance only needs to occur before an aircraft performs a flight. i.e., if an aircraft is grounded, even if the last A-type check was done 4 months ago, the aircraft only needs to perform an A-type maintenance check before it performs a flight, so if the next flight is due in 2 months, starting from the already passed 4 months, the aircraft only needs to perform the A-type maintenance check then, even if the last A-type check had been done 6 months before. To address this issue, the model could include some more constraints regarding if an aircraft is grounded or not, during specific periods of time. This way, if an aircraft is grounded longer than the threshold value of weeks between checks, the aircraft only needs to perform the maintenance activity when the aircraft becomes needed again, and only then.

5.5.3 Hangar facilities capacity availability

The purpose of this analysis is to show the impact of having different hangar facilities capacity availability and its influence on the total cost of the maintenance planning. The objective is to check if having more hangar capacity can help to reduce the total cost of the maintenance schedule, by reducing aircraft unavailability for doing maintenance checks earlier than possible, due to lack of hangar slots available.

To perform this analysis, the parameter $\boldsymbol{S_t}$ is given different values of 5, 6, 7, 8, 9 and 10, which represents the hangar capacity throughout the planning horizon. The value of 6 is already used in the case study, so the results presented are the same as the ones present in the subsection 5.3.2. Table 5.13 shows the results of this analysis.

Table 5.13 - Results for the hangar facilities capacity availability sensitivity analysis

S_t	Real minimum cost solution (k€)	Elapsed time (s)	Number of occurrences		
			Unavailability	A-type check	C-type check
5	-	-	-	-	-
6	52,290	2,886	510	463	47
7	51,890	3,008	502	455	47
8	51,690	2,951	498	451	47
9	51,660	2,758	498	450	47
10	51,540	2,936	495	448	47

For a hangar capacity of 5 aircrafts per week, the model cannot reach a feasible solution for the case study, which means that the minimum value of hangar facilities capacity must be at least 6 per week, considering the specifications and parameters of this case study.

From the other hangar facilities capacity values, a constant reduction on the minimum solution cost can be seen when the hangar availability increases. Although C-type checks cannot be avoided, there are A-type checks that can be avoided by increasing the hangar capacity, which means some A-type checks on the case study are being performed earlier than needed, consequently increasing the number of A-type checks throughout the planning horizon.

This analysis helps to interact with a possible framework that considers the workforce needed to perform such maintenance activities. A possible scenario, is where the hangar availability can change throughout the planning horizon, depending on the needs of the aircraft fleet and the workforce available, while analysing the cost impact of everything and taking informed decisions.

6 Conclusion

In the final section, main conclusions are emphasized, as well as limitations. Further research is also discussed.

6.1 Contributions

The main objective of this present dissertation is to elaborate an optimal aircraft maintenance schedule for the case study of TAP Air Portugal narrow-body fleet. Although a comprehensive real case study of maintenance scheduling could not be totally guaranteed for this dissertation, which made the comparison between schedules not possible in a fair manner. Thus, a comparison between the exact approach using the branch-and-bound method and a heuristic approach developed during this dissertation is studied and analysed. In fact, the main contribution of the present dissertation is the application of a heuristic approach to solve the aircraft maintenance scheduling problem. The comparison shows that the heuristic approach can achieve the same or quite similar values as the exact method, while requiring a much lower computational time. The study also shows that with an increasing number of planes, the reduction in computational time also increases, in terms of percentage, which means that using the heuristic approach for large-sized problems becomes more relevant.

Furthermore, a mixed-integer linear programming model is developed, which considers several technical and business aspects for the aviation industry. Regarding the technical aspects, the model considers not only the A-type and C-type maintenance checks and their limits/thresholds of flight hours, flight cycles and weeks between each type of maintenance check, which are set from the aircraft manufacturer, but it also considers the availability of the hangar and its capacity, as defined by the maintenance company of TAP Air Portugal, TAP M&E. Regarding the business aspects, the model takes into account the unavailability cost (or downtime cost) of the aircraft during the time it is scheduled for maintenance, and it also considers the influence of seasons in the aviation industry. Therefore, the model contributes with an additional penalty cost given for C-type maintenance checks that occur during the considered high season for the aviation industry, which includes summertime, and Christmas and new years' time, and thus avoiding C-type maintenance checks to occur during this period.

To sum up, the main contribution of this dissertation is to provide an improved scalable decision framework for optimizing aircraft maintenance scheduling, by solving the entire problem with a competitive computational time, using a heuristic approach, while considering all the airline company requirements and constraints, such as flight estimations and hangar facilities availability.

6.2 Limitations

Although the present model supplies a decision framework to support operations' teams to compute an optimized schedule for the aircraft maintenance activities, it still does not take into account every single aspect that influences the operations or the schedule. Some further steps and adaptations need to be considered in the future.

One of the limitations is not considering the necessary skilled workforce to perform each maintenance check type, i.e., the model considers the hangar availability only by the available space, but not the different number of technicians needed to execute such tasks, which can influence the hangar slots S_t available for each week. For this dissertation, this value was considered constant throughout the entire planning horizon, which could not represent a real-world situation in the aviation industry, due to variations on the workforce availability at each week.

Another limitation of the current model is the fact that in the first and last four weeks, not a single maintenance check is schedule for the case study. The way the model is formulated assumes as if the aircrafts have to perform a maintenance check at week 106, which means it will try to optimize the available limit interval, what will result in scheduling the last check as much further from the last week as possible. The third term of the objective function tries to minimize this effect, and that is the reason the first aircrafts to be scheduled occur nearer the planning horizon end, than the following aircrafts. Of course, when the slots available for these last weeks reach the maximum capacity, the model will schedule the last check nearer the end, but it still does not schedule it on the last four weeks. This limitation requires that the airline company should plan and run the model annually, in order to check for any changes for the maintenance schedule and adjust it accordingly.

Other limitation is that this model requires exact input data from the real-world, in order to be feasible and reliable. One of the main problems and limitations is the uncertainty associated with the prediction of flight hours and flight cycles for every week during the planning horizon, due to unexpected events (e.g. cancelled flights).

6.3 Future Research

For future research, several aspects can be improved, and new maintenance operations can be explored and integrated in the present decision framework.

One aspect that can be improved is the integration of the scheduling of skilled workforce, in order to create a more realistic and reliable aircraft maintenance scheduling. As previously mentioned, this can give a more precise input of the maintenance hangar availability, while considering costs associated with these operations. This way, the operational costs of maintenance can be more realistic, as it includes a wider range of the costs supported by aircraft maintenance.

Other improvement that can be implemented is the consideration of all maintenance check types, like the D-type, which is the heaviest maintenance check, and usually occurs between the interval of 6 to 10 years. Taking this into account also reflects on the need of increasing the scheduling planning horizon, which for the exact method could take an unreasonable computational time to schedule, but with the heuristic approach presented in this dissertation, is now less time consuming to achieve an optimal and feasible solution. Besides, considering other aircrafts of the fleet, such as the wide-body aircrafts, can give a broader look on the entire maintenance operations of airline companies, and should be further researched and implemented.

Furthermore, a study on the routing problem should be considered, as it can support the airline companies to consider more hangar facilities and provide a higher cost reduction on maintenance operations. For this specific case study, taking into account the routing could take advantage of the two maintenance facilities in Brazil, run by TAP M&E Brazil. This means more hangar slots available, which can reduce the need of aircrafts performing maintenance checks early due to lack of hangar availability.

Lastly, an improvement that could be studied is the consideration of eventual discrepancies on the estimation of weekly flight hours or flight cycles, because of cancelled flights or any other events. Although this could prove to be a difficult task to implement, analysing previous data from each aircraft to assess an average value of divergence between estimations and what was observed, could be an option to take into account unexpected events throughout the planning horizon.

In conclusion, the previous proposals are research opportunities, which can help improve the decision framework for airline companies to reduce their aircrafts fleet maintenance operational costs. Although the aviation sector is going through deep changes, due to the impact of the current pandemic, maintenance operations optimization is still a topic that will need further research, even after the expected recovery by the aviation industry in a post-pandemic phase.

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8 Appendix

A1 – Airbus A319 aircrafts fleet description for the Case Study

Plane number	Aircraft type	Registration
1	Airbus A319	CS-TTA
2	Airbus A319	CS-TTB
3	Airbus A319	CS-TTC
4	Airbus A319	CS-TTD
5	Airbus A319	CS-TTE
6	Airbus A319	CS-TTF
7	Airbus A319	CS-TTG
8	Airbus A319	CS-TTH
9	Airbus A319	CS-TTI
10	Airbus A319	CS-TTJ
11	Airbus A319	CS-TTK
12	Airbus A319	CS-TTL
13	Airbus A319	CS-TTM
14	Airbus A319	CS-TTN
15	Airbus A319	CS-TTO
16	Airbus A319	CS-TTP
17	Airbus A319	CS-TTQ

18	Airbus A319	CS-TTR
19	Airbus A319	CS-TTS
20	Airbus A319	CS-TTU
21	Airbus A319	CS-TTV

A2 – Airbus A320 aircrafts fleet description for the Case Study

Plane number	Aircraft type	Registration
22	Airbus A320	CS-TMW
23	Airbus A320	CS-TNG
24	Airbus A320	CS-TNH
25	Airbus A320	CS-TNI
26	Airbus A320	CS-TNJ
27	Airbus A320	CS-TNK
28	Airbus A320	CS-TNL
29	Airbus A320	CS-TNM
30	Airbus A320	CS-TNN
31	Airbus A320	CS-TNP
32	Airbus A320	CS-TNQ
33	Airbus A320	CS-TNR

34	Airbus A320	CS-TNS
35	Airbus A320	CS-TNT
36	Airbus A320	CS-TNU
37	Airbus A320	CS-TNV
38	Airbus A320	CS-TNW
39	Airbus A320	CS-TNX
40	Airbus A320	CS-TNY
41	Airbus A320	CS-TQD

A3 – Airbus A321 aircrafts fleet description for the Case Study

Plane number	Aircraft type	Registration
42	Airbus A321	CS-TJE
43	Airbus A321	CS-TJF
44	Airbus A321	CS-TJG
45	Airbus A321	CS-TJH

A4 – Accumulated flight hours, flight cycles and weeks for each A319 aircraft of the Case Study

Plane number	Parameters					
	$FH^0_{1,p}$	$FC^0_{1,p}$	$W^0_{1,p}$	$FH^0_{2,p}$	$FC^0_{2,p}$	$W^0_{2,p}$
1	515	255	8	2,612	1,296	38
2	162	82	3	825	390	13
3	684	344	11	684	344	11
4	648	306	11	0	0	0
5	222	108	4	4,060	2,019	62
6	150	64	3	150	64	3
7	351	164	6	3,756	1,808	55
8	154	66	3	4,084	2,004	60
9	455	216	2	3,180	1,552	46
10	476	252	8	1,161	583	17
11	114	62	3	298	172	5
12	54	27	1	6,137	3,098	94
13	534	254	8	6,645	3,316	99
14	659	296	10	4,689	2,275	70
15	273	122	5	4,377	2,134	65
16	23	9	1	3,442	1,684	50

17	589	269	9	4,516	2,082	67
18	233	108	4	6,561	3,248	98
19	384	169	6	6,460	3,120	94
20	573	281	9	6,220	2,999	91
21	258	108	4	6,095	2,914	89

A5 – Accumulated flight hours, flight cycles and weeks for each A320 aircraft of the Case Study

Plane number	**Parameters**					
	$FHo_{1,p}$	$FCo_{1,p}$	$Wo_{1,p}$	$FHo_{2,p}$	$FCo_{2,p}$	$Wo_{2,p}$
22	250	98	4	5,013	1,981	65
23	530	219	8	3,862	1,699	53
24	24	10	1	2,600	1,153	36
25	577	247	13	558	235	8
26	339	155	5	4,314	1,965	60
27	180	81	3	2,836	1,284	39
28	495	201	7	2,551	1,133	35
29	539	225	8	0	0	0
30	33	9	1	6,698	2,868	92
31	446	165	6	5,453	2,093	68

32	10	3	1	667	233	8
33	611	231	8	611	231	8
34	9	5	1	9	5	1
35	221	93	3	221	93	3
36	230	85	6	0	0	0
37	256	97	4	7,171	2,761	89
38	333	123	5	3,526	1,352	42
39	126	48	2	126	48	2
40	96	37	2	2,237	915	36
41	0	0	0	1,427	536	18

A6 – Accumulated flight hours, flight cycles and weeks for each A321 aircraft of the Case Study

Plane number	**Parameters**					
	$FHo_{1,p}$	$FCo_{1,p}$	$Wo_{1,p}$	$FHo_{2,p}$	$FCo_{2,p}$	$Wo_{2,p}$
42	94	42	2	3,840	1,582	50
43	336	143	5	1,149	477	16
44	431	189	7	3,630	1,483	48
45	166	64	3	2,818	1,172	55

A7 – Average of the values of total flight hours on the week before an A-type and C-type maintenance check occurs and the average accumulation of weeks on the week before a C-type maintenance occurs for the heuristic approach of the case study

Plane	Post-Processing Parameters		
	$TFH_{1,p,t}$	$TFH_{2,p,t}$	$TW_{2,p,t}$
	Average (hours)	**Average (hours)**	**Average (weeks)**
A319 [1]	717.6	5,938.5	85
A319 [2]	685.4	1,926.5	29
A319 [3]	690.3	4,609	67
A319 [4]	658.9	4,267	61
A319 [5]	692.1	6,838	101
A319 [6]	676.3	4,557	66
A319 [7]	621.5	6,534	94
A319 [8]	676.7	6,862	99
A319 [9]	632.2	6,706	96
A319 [10]	669.9	5,086	73
A319 [11]	672.3	5,352.5	77
A319 [12]	704	6,815	104
A319 [13]	682.3	6,844.5	102
A319 [14]	624.6	5,864	87

A319 [15]	589.6	6,875	100
A319 [16]	630.2	7,167.5	103
A319 [17]	618.9	7,014	102
A319 [18]	585.9	6,960	104
A319 [19]	606	6,659.5	97
A319 [20]	622.9	6,898	101
A319 [21]	594.2	6,773	99
A320 [22]	656.9	7,458.5	95
A320 [23]	682.9	7,210.5	94
A320 [24]	700.1	6,497	84
A320 [25]	687.2	5,225	66
A320 [26]	665.2	7,421	98
A320 [27]	650.9	6,810	88
A320 [28]	679.7	6,602	85
A320 [29]	690.7	4,827	60
A320 [30]	693.6	5,923.5	78
A320 [31]	668.1	7,058.5	88
A320 [32]	690.9	5,257	65
A320 [33]	683.1	5,278	66
A320 [34]	698.6	4,676	59

A320 [35]	647.1	4,967.5	62
A320 [36]	648.2	4,907.5	61
A320 [37]	664.8	6,041	75
A320 [38]	671.8	7,035.5	85
A320 [39]	702.5	4,716	59
A320 [40]	699.5	6,057	83
A320 [41]	690.3	5,863	73
A321 [42]	680.5	7,257.5	93
A321 [43]	697.3	5,380	70
A321 [44]	706.8	7,126	92
A321 [45]	679.6	6,534.5	102

A8 – Maintenance schedule solution of the heuristic approach for the Case Study

		Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Nº	Type	Registration	5 feb.	12 feb.	19 feb.	26 feb.	5 mar.	12 mar.	19 mar.	26 mar.	2 apr.	9 apr.	16 apr.	23 apr.	30 apr.	7 may	14 may	21 may	28 may	4 jun.	11 jun.	18 jun.	25 jun.	2 jul.	9 jul.	16 jul.	23 jul.	30 jul.
1	A319	CS-TTA					A2											A3										
2	A319	CS-TTB										A4								A1 + C2								
3	A319	CS-TTC		A2											A3											A4		
4	A319	CS-TTD			A2											A3											A4	
5	A319	CS-TTE									A4											A1						
6	A319	CS-TTF										A1											A2					
7	A319	CS-TTG								A1									A2							A3		
8	A319	CS-TTH										A4										A1						
9	A319	CS-TTI						A3											A4									
10	A319	CS-TTJ						A3										A4										
11	A319	CS-TTK											A1											A2				
12	A319	CS-TTL												A2 + C12											A3			
13	A319	CS-TTM					A2 + C12											A3										
14	A319	CS-TTN			A2										A3						A4 + C12							
15	A319	CS-TTO									A3										A4							
16	A319	CS-TTP												A2											A3			
17	A319	CS-TTQ				A2											A3											A4
18	A319	CS-TTR								A2 + C10											A3							
19	A319	CS-TTS					A2 + C10									A3											A4	
20	A319	CS-TTU				A2								A3 + C10											A4			
21	A319	CS-TTV					A2							A3 + C10											A4			
22	A320	CS-TMW								A1										A2								A3
23	A320	CS-TNG				A4										A1										A2		
24	A320	CS-TNH											A4										A1					
25	A320	CS-TNI				A4										A1										A2		
26	A320	CS-TNJ							A1										A2								A3	
27	A320	CS-TNK									A3										A4							
28	A320	CS-TNL					A3										A4									A1		
29	A320	CS-TNM				A2										A3										A4		
30	A320	CS-TNN											A1 + C10										A2					
31	A320	CS-TNP					A3										A4							A1 + C9				
32	A320	CS-TNQ											A2										A3					
33	A320	CS-TNR			A4										A1										A2			
34	A320	CS-TNS											A3										A4					
35	A320	CS-TNT								A2										A3								A4
36	A320	CS-TNU								A2										A3								
37	A320	CS-TNV						A2 + C5											A3									A4
38	A320	CS-TNW							A4										A1									A2
39	A320	CS-TNX										A2										A3						
40	A320	CS-TNY										A1										A2						
41	A320	CS-TQD											A2										A3					
42	A321	CS-TJE										A3										A4						
43	A321	CS-TJF							A2										A3									
44	A321	CS-TJG						A2										A3										A4
45	A321	CS-TJH									A3										A4							

		Week	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
Nº	Type	Registration	6 aug.	13 aug.	20 aug.	27 aug.	3 sep.	10 sep.	17 sep.	24 sep.	1 oct.	8 oct.	15 oct.	22 oct.	29 oct.	5 nov.	12 nov.	19 nov.	26 nov.	3 dec.	10 dec.	17 dec.	24 dec.	31 dec.	7 jan.	14 jan.	21 jan.	28 jan.
1	A319	CS-TTA	A4											A1											A2 + C2			
2	A319	CS-TTB			A2										A3											A4		
3	A319	CS-TTC									A1											A2						
4	A319	CS-TTD										A1									A2							
5	A319	CS-TTE					A2										A3 + C1											
6	A319	CS-TTF						A3											A4									
7	A319	CS-TTG						A4									A1 + C1											
8	A319	CS-TTH					A2										A3 + C1											
9	A319	CS-TTI		A1							A2									A3								A4+C1
10	A319	CS-TTJ	A1											A2										A3				
11	A319	CS-TTK						A3										A4										
12	A319	CS-TTL								A4										A1								
13	A319	CS-TTM	A4											A1										A2				
14	A319	CS-TTN					A1										A2										A3	
15	A319	CS-TTO		A1									A2 + C12										A3					
16	A319	CS-TTP								A4										A1								
17	A319	CS-TTQ											A1 + C1											A2				
18	A319	CS-TTR			A4										A1									A2				
19	A319	CS-TTS									A1											A2						
20	A319	CS-TTU								A1										A2								
21	A319	CS-TTV								A1										A2								
22	A320	CS-TMW						A4 + C11											A1									
23	A320	CS-TNG							A3										A4 + C1									
24	A320	CS-TNH				A2										A3										A4 + C1		
25	A320	CS-TNI							A3										A4									A1
26	A320	CS-TNJ							A4							A1 + C12											A2	
27	A320	CS-TNK		A1									A2								A3						A4 + C12	
28	A320	CS-TNL							A2										A3									A4 + C12
29	A320	CS-TNM							A1										A2									
30	A320	CS-TNN				A3										A4									A1			
31	A320	CS-TNP						A2										A3										A4
32	A320	CS-TNQ				A4										A1									A2			
33	A320	CS-TNR						A3										A4										A1
34	A320	CS-TNS				A1										A2										A3		
35	A320	CS-TNT									A1										A2							
36	A320	CS-TNU	A4									A1										A2						
37	A320	CS-TNV									A1										A2							
38	A320	CS-TNW									A3										A4 + C8							
39	A320	CS-TNX			A4										A1										A2			
40	A320	CS-TNY			A3										A4										A1 + C4			
41	A320	CS-TQD				A4										A1								A2				
42	A321	CS-TJE				A1									A2						A3 + C12							
43	A321	CS-TJF	A4										A1										A2					
44	A321	CS-TJG										A1										A2 + C11						
45	A321	CS-TJH			A1										A2										A3 + C9			

		Week	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78
Nº	Type	Registration	4 feb.	11 feb.	18 feb.	25 feb.	4 mar.	11 mar.	18 mar.	25 mar.	1 apr.	8 apr.	15 apr.	22 apr.	29 apr.	6 may	13 may	20 may	27 may	3 jun.	10 jun.	17 jun.	24 jun.	1 jul.	8 jul.	15 jul.	22 jul.	29 jul.
1	A319	CS-TTA									A3											A4						
2	A319	CS-TTB										A1											A2					
3	A319	CS-TTC						A3 + C2											A4									
4	A319	CS-TTD		A3									A4 + C1											A1				
5	A319	CS-TTE		A4											A1											A2		
6	A319	CS-TTF			A1										A2 + C2											A3		
7	A319	CS-TTG		A2											A3										A4			
8	A319	CS-TTH		A4										A1								A2						
9	A319	CS-TTI	A4+C1												A1										A2			
10	A319	CS-TTJ						A4 + C1												A1								
11	A319	CS-TTK	A1											A2										A3 + C1				
12	A319	CS-TTL				A2											A3											A4
13	A319	CS-TTM						A3											A4									
14	A319	CS-TTN										A4										A1						
15	A319	CS-TTO			A4									A1										A2				
16	A319	CS-TTP			A2 + C12											A3										A4		
17	A319	CS-TTQ						A3									A4									A1		
18	A319	CS-TTR					A3										A4									A1		
19	A319	CS-TTS				A3										A4								A1				
20	A319	CS-TTU				A3										A4											A1	
21	A319	CS-TTV				A3										A4									A1			
22	A320	CS-TMW	A2										A3										A4					
23	A320	CS-TNG	A1										A2										A3					
24	A320	CS-TNH								A1										A2								
25	A320	CS-TNI								A2 + C1											A3							
26	A320	CS-TNJ									A3										A4							
27	A320	CS-TNK										A1										A2						
28	A320	CS-TNL	A4 + C12										A1										A2					
29	A320	CS-TNM	A3									A4 + C10											A1					
30	A320	CS-TNN						A2									A3 + C11										A4	
31	A320	CS-TNP										A1										A2						
32	A320	CS-TNQ							A3 + C6										A4									A1
33	A320	CS-TNR								A2 + C6											A3							
34	A320	CS-TNS								A4 + C6										A1								
35	A320	CS-TNT		A3							A4 + C6										A1							
36	A320	CS-TNU			A3								A4 + C5										A1					
37	A320	CS-TNV			A3									A4 + C6										A1				
38	A320	CS-TNW			A1										A2									A3				
39	A320	CS-TNX							A3 + C8										A4									A1
40	A320	CS-TNY							A2										A3									A4
41	A320	CS-TQD					A3 + C1											A4									A1	
42	A321	CS-TJE					A4										A1										A2	
43	A321	CS-TJF				A3 + C12										A4									A1			
44	A321	CS-TJG					A3										A4										A1	
45	A321	CS-TJH							A4										A1									A2

		Week	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Nº	Type	Registration	5 aug.	12 aug.	19 aug.	26 aug.	2 sep.	9 sep.	16 sep.	23 sep.	30 sep.	7 oct.	14 oct.	21 oct.	28 oct.	4 nov.	11 nov.	18 nov.	25 nov.	2 dec.	9 dec.	16 dec.	23 dec.	30 dec.	6 jan.	13 jan.	20 jan.	27 jan.	3 fev.
1	A319	CS-TTA					A1											A2											
2	A319	CS-TTB						A3											A4										
3	A319	CS-TTC		A1										A2									A3						
4	A319	CS-TTD							A2										A3										
5	A319	CS-TTE							A3										A4										
6	A319	CS-TTF							A4									A1											
7	A319	CS-TTG							A1									A2											
8	A319	CS-TTH					A3											A4											
9	A319	CS-TTI							A3									A4											
10	A319	CS-TTJ			A2										A3								A4						
11	A319	CS-TTK						A4										A1											
12	A319	CS-TTL										A1											A2						
13	A319	CS-TTM		A1										A2										A3					
14	A319	CS-TTN				A2									A3									A4					
15	A319	CS-TTO					A3									A4								A1					
16	A319	CS-TTP						A1								A2								A3					
17	A319	CS-TTQ						A2								A3								A4					
18	A319	CS-TTR			A2										A3									A4					
19	A319	CS-TTS			A2											A3									A4				
20	A319	CS-TTU								A2							A3								A4				
21	A319	CS-TTV						A2									A3								A4				
22	A320	CS-TMW				A1								A2								A3							
23	A320	CS-TNG				A4								A1								A2							
24	A320	CS-TNH	A3									A4									A1								
25	A320	CS-TNI		A4									A1									A2							
26	A320	CS-TNJ		A1									A2									A3							
27	A320	CS-TNK			A3									A4								A1							
28	A320	CS-TNL				A3								A4								A1							
29	A320	CS-TNM				A2									A3								A4						
30	A320	CS-TNN								A1										A2									
31	A320	CS-TNP			A3								A4								A1								
32	A320	CS-TNQ									A2									A3									
33	A320	CS-TNR		A4									A1								A2								
34	A320	CS-TNS	A2									A3									A4								
35	A320	CS-TNT		A2									A3								A4								
36	A320	CS-TNU			A2								A3								A4								
37	A320	CS-TNV				A2									A3								A4						
38	A320	CS-TNW					A4								A1								A2						
39	A320	CS-TNX									A2									A3									
40	A320	CS-TNY									A1									A2									
41	A320	CS-TQD								A2										A3									
42	A321	CS-TJE									A3									A4									
43	A321	CS-TJF							A2										A3										
44	A321	CS-TJG								A2									A3										
45	A321	CS-TJH									A3								A4										